Yang Hui

This entry features the mathematician **Yang Hui** (c. 1238 – c. 1298) but also is a brief account of the development of Chinese mathematics. Chinese civilizations along the Changjiang (Yangtze) and the Huang He (Yellow) rivers are comparable in ages with those along the Nile or between the Tigris and the Euphrates. Fairly reputable tradition tells us that the Chinese first formulated some theories about descriptive astronomy around the reign of Fuh-hi, the first emperor of China (2852 – 2738 BCE). The origins of Chinese mathematics can be traced to at least the third millennium BCE, when numeration systems and a smattering of geometry were developed. The bible of Chinese civilization *The Book of Changes* (*Yi jing*), written in the 12th century BCE, contained instruction for making calculations. The Book of Changes used hexagrams to describe the interaction between *Yin* and *Yang*. When Yin and Yang are balanced, one achieves the state of *Taiji*, the Supreme Ultimate.

Little is known of ancient Chinese mathematics, since during the Ch′in dynasty the despotic emperor Shih Huang-ti, who ruled from 221 to 207 BCE, ordered all books to be burned and scholars buried alive. The oldest known truly mathematical texts were produced during the following Han period in the third century BCE by scholars who transcribed China's literary and scientific traditions from memory or from remaining fragments of scroll. Although chiefly dealing with the calendar, the *Chóu-peï Suan-king*, (*The Arithmetic Classic of the Gnomon and the Circular Paths of Heaven*) also contained various modern mathematical principles. It included the oldest known proof of the Pythagorean Theorem. *Jiuzhang suanshu* or *Chu Chang Suan Shu* (*Nine Chapters of the Mathematical Arts* c. 202 BCE) was the greatest of the Chinese mathematical classics. It contained 246 problems divided into nine chapters,

covering topics such as surveying, weights and measures, currency and tax collection, percentage and proportion, the rule of three, the rule of false position, and simultaneous linear equations.

The next major work was the seventh century A.D. textbook *Suanjing shi shu* (*The Ten Computational Canons)*, a compilation of the most important works then known. In the thirteenth century the algebras *Shu-shu chiu-chang* of Ch'in Chiu-shao and Li Chih's *Ts'e-yüan hai-ching* appeared. These were followed by the work of Yang Hui, a minor civil servant. He wrote several mathematics books, including *Hsiang-chieh chiu-chang suan-fa* (*Detailed Analysis of the Mathematical Rules in the Nine Chapters*) and the three volumes *Ch'eng-ch'u t'ung-pien pen-mo* (*Fundamental Mutual Changes in Multiplications and Divisions*).

The first book is a revision of the *Nine Chapters*, incorporating new material and discarding some unwanted sections. Yang Hui added three chapters to the original nine. According to the preface, he selected 80 of the original 246 problems for detailed discussion. The introduction to the book as well as the first two chapters are now lost. The first chapter dealt with ordinary methods of multiplication and division, and the second with "Surveying the Land." Titles of some of the other chapters are: "Excess and Deficiency," and "Right Angles." The first volume of Yang Hui's second work was *Fundamental Mutual Changes in Multiplications and Divisions*. The other two volumes were *Treasure of Mathematical Arts on the Mutual Changes in Multiplications and Divisions*, and *Fundamentals of the Applications of Mathematics*. Yang Hui also wrote the two-volume *Practical Rules of Mathematics for Surveying* and *Continuation of Ancient Mathematical Methods for Elucidating the Strange Properties of Numbers*. Subsequently, all seven of Yang Hui's treatises were incorporated as one work, *The Mathematical Arts of Yang Hui*. It was first printed in 1378 in China and in Korea in 1433.

In *Practical Rules*, Yang Hui gave the earliest explanations of the Chinese methods for solving

quadratic equations, including the familiar quadratic formula. The *Strange Properties of Numbers* is the first extant Chinese text to include magic squares (or "vertical and horizontal diagrams" as they were called) of order greater than three. According to legend, as the Sage King Yu the Great (died 2197 BCE) stood on the bank of the river Luo a tortoise emerged from the Luo. On the undershell of the tortoise was a 3 x 3 array of the numbers 1 through 9 encoded in dots. This was the Luo Shu [Figure 11.1*a*], a unique perfect magic square of order three. Each row, column and diagonal has a sum of 15. Daoists thought of the odd numbers in the central cross of the Luo Shu as yang numbers and the even numbers as yin numbers. The Luo Shu represented a state of balance and harmony between the numbers. Emperor Yu received yet another magic square gift from a miraculous animal, this time a dragon horse which emerged from the Yellow River. The horse carried what has come to be known the *Ho Thu* [Figure 11*b*]. The arrangement of this diagram is a bit different. If the two center sets of dots representing 5 and 10 are ignored, both the even and odd sets add up to 20.

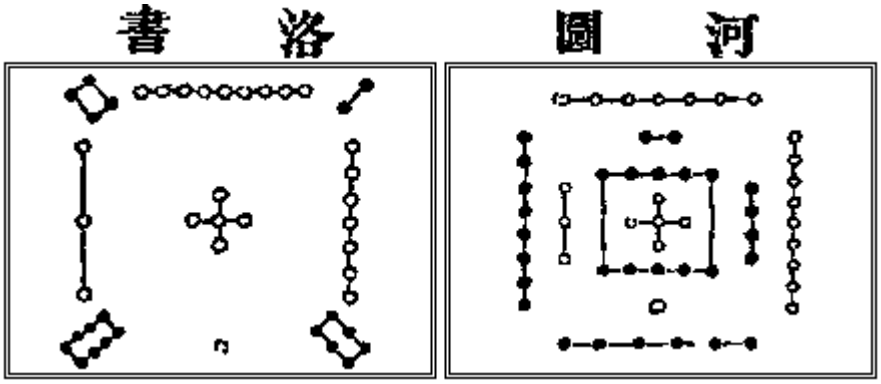

Figure 11.1*a* Figure 11.1*b*

Yang Hui considered magic squares of orders up to nine and fascinating interlocking magic circles such as that in Figure 11.2. In a magic circle, *n* concentric circles are cut by *n* diameters, and numbers are placed on the points of intersection. In Figure 11.2, the sum of the numbers of any diameter is equal to 147 and that of the circles is 138.

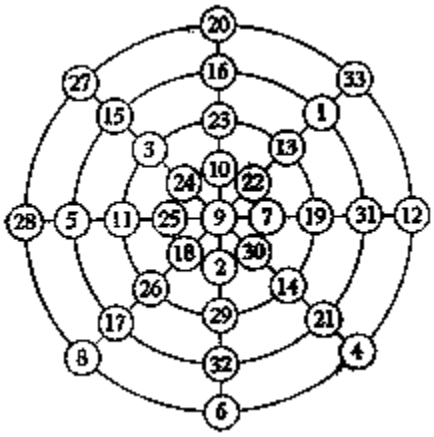

Figure 11.2

Yang Hui used the method of successive approximations for finding square and cube roots. In 1819, W.G. Horner expounded these methods, originally found in the *Nine Chapters,* unaware that he had rediscovered methods already a thousand years old. The geometric technique for extracting roots is equivalent to using the binomial expansion, whose numerical coefficients can be expressed by what is now known as “Pascal’s Triangle.” This pattern was known and used in calculations in China for several centuries before it was known in the West. In China, it is known as “Yang Hui’s triangle.” The frontispiece of the *Siyan yujian* (1303) by Zhu Siyuan shows this triangle of coefficients written in Chinese characters [Figure 11.3].

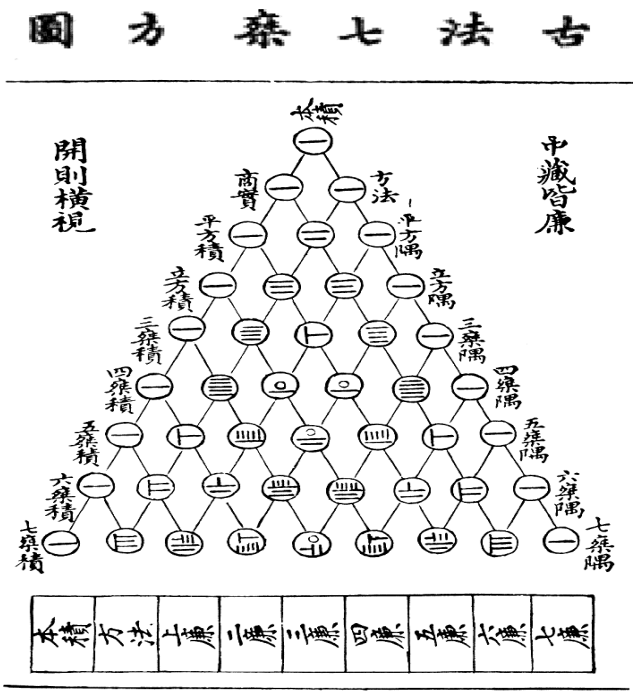

Figure 11.3

The ancient Chinese used two different numeration systems, one employed a multiplicative principal and the other a positional one. Both were essentially decimal. In the first, distinct symbols were used for the digits from one to ten and for powers of ten. When writing a number, the digits were multiplied by their place values. For instance, the number 1725 would be represented by the symbol for one, followed by the symbol for one thousand, then the symbol for seven, followed by the symbol for one hundred, then the symbol for two, followed by the symbol for ten, and finally five. The word "ling" (which means "zero") or its symbol is mentioned whenever any power of ten is not represented in the expression of the number. But this custom was established late in the development of the Chinese-number system.

In China's system of "rod numerals", the digits from one to nine are represented in the first row of

Figure 11.4 and the first nine multiples of ten in the second row.

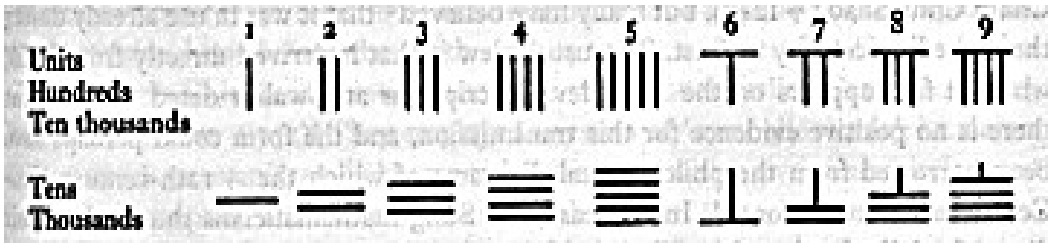

Figure 11.4

By the use of these eighteen symbols in positions from right to left, numbers of any practical value could be represented. The round symbol for zero appeared only much later. Figure 11.5 gives the symbols for 1 through 10, 100, and 1000 in terms of Chinese characters, which respectively represent the words yi (one), er (two), san (three), si (four), wu (five), liu (six), qi (seven), ba (eight), jiu (nine), shi (ten), pai (100), and ch′ien (thousand).

Figure 11.5

Quotation of the Day: “Units are vertical, tens are horizontal,

Hundreds stand, thousands lie down,

Thus thousands and tens look the same,

Ten thousands and hundreds look alike.”

-Sun Tsu (200 CE) explains the method of recording numbers.