Hermann Weyl

Arguably the greatest mathematician of his generation, **Hermann Klaus Hugo Weyl** (November 9, 1885 – December 8, 1955) explored mathematics, physics, and philosophy with equal vigor. Known as Peter to his close friends, he was born at Elmshorn, near Hamburg in Germany, the son of Ludwig and Anna Weyl. He attended the Gymnasium at Altona, where as a schoolboy he was quite taken by Immanuel Kant's doctrine in *Critique on Pure Reason, "*that space and time are not inherent, in the objects of the world, existing as such and independently of our awareness, but are, rather, *conceptual forms* or *intuitions* based on our intellects." Weyl entered the University of Göttingen in 1903, where he read David Hilbert's *Foundations of Geometry*, which made Kant's "bondage to Euclidean geometry" appear naïve, causing Weyl to abandoned Kantian philosophy. He spent one year studying at the University of Munich but was awarded a doctorate from Göttingen in 1908 for a thesis supervised by Hilbert. Weyl was a *Privatdozent* at Göttingen for the next few years but declined an offer to succeed Felix Klein when he retired in 1913. Instead he accepted the chair of mathematics at the Technische Hochschule in Zurich, Switzerland, where he stayed until 1930.

At Zurich he married Helene Joseph, a translator of Spanish literature. They had two sons, one of whom, Fritz Joachim received a Ph.D. in mathematics from Princeton University in 1939. Helene died in 1948 and two years later Weyl married Ellen Bär of Zurich. While at the Technische Hochschule, Weyl worked with Einstein, who interested the younger man in the mathematics of relativity and Riemannian geometry. Weyl came to believe erroneously that he had found a way to unify gravitation and electromagnetism geometrically by extending Einstein's theory of general relativity. Although his theory met with considerable resistance, his mathematical structure survived and reappeared over the years in different guises, independent of Weyl's original formulation. Weyl succeeded Hilbert in 1930, and following the purge of Jewish mathematicians in 1933, briefly served as Director of the university's famous Mathematical Institute. His efforts to salvage mathematics at Göttingen from the Nazis being unsuccessful, he decided he couldn't abide living under Hitler's rule, and prepared to leave Germany. He wrote in his memoirs:

> "I could not bear to live under the rule of that demon [Hitler] who had dishonored the name of Germany and although the wrench was hard and the mental agony so cruel that I suffered a severe breakdown; I shook the dust of the fatherland from my feet."

Weyl settled in the United States as research professor in mathematical physics at the Institute for Advanced Study at Princeton University, where he remained until his retirement. He spent the last four years of his life traveling between Zurich and Princeton, dying at Zurich in 1955.

Before leaving Göttingen for Zurich, Weyl immersed himself in the mathematics of Riemann. His goal was to do for Riemann's function theory what Hilbert had done for Euclidean geometry, that is, give it an axiomatic and rigorous basis. The result was his classic book *Die Idée der Riemannschen Fläche* (*The Concept of a Riemann Surface,* 1913), in which he created a new branch of mathematics by uniting analysis, geometry and topology, which inspired all later developments on the theory of differential and complex manifolds. Weyl produced the first unified field theory for which the Maxwell electromagnetic field and the gravitational field appear as geometrical properties of space-time. His results appeared in his book *Raum-Zeit-Materie* (*Space-Time-Matter*, 1918). Weyl's research interest in group theory and Hilbert spaces led him to develop techniques that proved central to the rapidly evolving theory of quantum mechanics and the unification of matrix mechanics and wave mechanics. Weyl evolved the concept of continuous groups using matrix representations, showing how symmetry relates group theory and continuous groups, and how this powerful tool could be used in solving quantum mechanical problems. He expounded his ideas in *Gruppentheorie und Quantenmechanik* (*Group Theory and Quantum Mechanics,* 1928).

Besides those already mentioned, Weyl's lucidly written treatises include: *The Classical Groups* (1939), *Algebraic Theory of Numbers* (1940), *Philosophy of Mathematics and Natural Science* (1949), and his masterly merging of art and mathematics, *Symmetry* (1952). In reviewing the latter work, *Scientific American* reported: "Dr. Weyl presents a masterful and fascinating survey of the applications of the principle of symmetry in sculpture, architecture, ornament, and design; its manifestations in organic and inorganic nature; and its philosophical and mathematical significance." In addition to his outstanding mathematical work, Weyl's was also deeply interested in philosophy, logic, and the history of mathematics. His literary style was almost poetic. For example, in his introduction to *The Classical Groups*, he commented on his enforced transition from writing in German to English: "the gods have imposed upon my writing the yoke of a foreign language that was not sung at my cradle."

In the quest to give all mathematics an axiomatic basis and to rigorously prove theorems, classical logic was viewed as something that preceded mathematics and judged it. This was fine until logic was applied to infinite sets, which led to troubling antinomies. Weyl likened this as the original sin that led to the Fall of mathematics and felt that the occurrences of contradictions was not what was surprising, but that they showed up so late in the game. The paradoxes led to the establishment of schools of logicians, mathematicians, and philosophers intent on healing mathematics' wounds. Weyl was a member of the intuitionist school of Dutch mathematician L.E.J. Brouwer. Intuitionist proponents believed that in the realm of logic there are certain clear, *intuitively* acceptable logical principles or procedures that can be used to assert new theorems from old ones. These principles are part of the fundamental mathematical intuition. The intuitionists analyzed which logical principles are allowable and which have been applied too freely. They believed this was the law of the excluded middle, which asserts that every meaningful statement is either true or false, and not both. This law is the basis of all indirect proofs. The denial of the excluded middle law gives rise to a new possibility, undecidable propositions. The intuitionists rejected nonconstructive proofs, that is, those that proved the existence of something without showing how to construct it. Weyl said nonconstructive proofs "inform the world that a treasure exists without disclosing its location."

Quotation of the Day: "Logic is the hygiene the mathematician practices to keep his ideas healthy and strong." – Hermann Weyl