Evangelista Torricelli

In his brief life, the interests of Italian mathematician and physicist **Evangelista Torricelli** (October 15, 1608 – October 25, 1647) ranged from pure mathematics to experimental physics. He worked on the quadrature of the cycloid and the conics and found the length of the logarithmic spiral. Along with his mentor Galileo, Torricelli pioneered telescopic astronomy. He invented the mercury barometer and discovered the effect of atmospheric pressure on water in a suction pump and was the first person to produce a sustained vacuum. He found the value of gravity by observing the motion of two weights connected by a string passing over a fixed pulley.

Born in Faenza, Torricelli was the eldest of three children of a poor textile artisan. His parents sent him to live with his uncle, a Camaldolese monk, who supervised his nephew's education. Torricelli took courses in mathematics and philosophy with the Jesuits in Faenza. He showed such aptitude that the Benedictine Benedetto Castelli of the University of Sapienza in Rome took him as a private student. Castelli, who was a former student of Galileo, thought so highly of Torricelli's genius that he made him his secretary, a post the younger man held between 1626 and 1632. During the following nine years Torricelli served as secretary of Monsignor Giovanni Ciampoli, Galileo's friend and protector. For three months in 1641 and 1642 Torricelli was Galileo's secretary, succeeding the latter as the court mathematician to the Grand Duke of Tuscany upon Galileo's death. He was also appointed to a long-vacant chair of mathematics at the "Studio fiorentino." Torricelli held these posts until his death at the ducal palace in Florence.

Torricelli's 1644 discovery of atmospheric pressure was the result of his attempt to solve the problem of why water could not be pumped from a mine or a well more than 32 feet deep. He deduced that the reason was that the atmosphere possessed weight and therefore exerted a pressure. He verified the idea by filling a tube three-feet long, known as a *Torricelli tube* [Figure 10.3], with quicksilver (mercury), a liquid fourteen times as dense as water, and hermetically sealed it at one end with the open end in a basin of mercury. He found that with each repeat of the experiment the height of the mercury column fell to about 30 inches (760 mm), leaving an empty space above it. Torricelli reasoned that his empty space was a vacuum [*Torricelli vacuum*] and the weight of the column was being balanced by the weight of the atmosphere. He later noticed small daily variations in the height of the column, which he attributed to changes in the weather. Thus Torricelli gave the first description of a barometer [Figure 10.4], the basis of the science of meteorology. Because atmospheric pressure changes with the distance above or below sea level, a barometer also can be used to measure altitude. Normal atmospheric pressure is about 14.7 pounds per square inch, equivalent to 29.92 inches or 760 mm of mercury. An aneroid barometer, that is, a clock face type barometer [Figure 10.5] indicates pressure on a dial using a needle that is mechanically linked to a partially evacuated chamber, which responds to pressure changes.

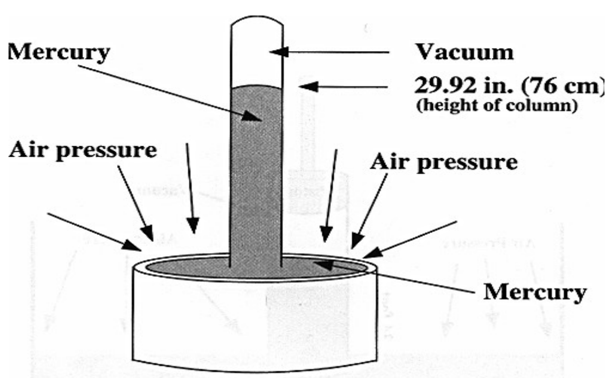

Figure 10.3

Figure 10.4

Figure 10.5

It may be difficult to fathom how atmospheric pressure could be controversial, but in Torricelli's time it was. In building his barometer, he concluded that the space above the mercury in his glass tube was a vacuum. However, Aristotle had insisted, "Nature abhors a vacuum and to the Jesuits of Torricelli's time Aristotle was an authority not to be questioned. Had he pushed the idea too hard, he might have suffered the fate of his master Galileo, but Torricelli was a cautious man and avoided trouble. He merely described the experiment but took no position in the philosophical argument concerning the existence of a vacuum. In a letter to Michelangelo Ricci, he tactfully wrote that, "Many have said that the vacuum does not exist, others that it can exist but only with difficulty and against the repugnance of Nature."

Torricelli was greatly interested in astronomy and was a committed Copernican. However, when Galileo was condemned in 1633, Torricelli prudently gave up further work in astronomy. In mathematics, he took delight with problems involving the use of infinitesimal methods. He was a mathematician of considerable talent, who used Cavalieri's method of indivisibles to solve the famous problem of finding the area under one arch of the cycloid, and later determined the length of the infinitely many revolutions of the logarithmic spiral ($\rho = e^{\theta}$). He also applied the geometry of indivisibles to the determination of the center of gravity of figures. Torricelli proved that the body which he called "the acute hyperbolic solid" [Figure 10.6], which is generated by revolving $y = 1/x$ about the x-axis from $x = 1$ to $x = \infty$ has finite volume. Clearly this surprising result for the infinitely long solid, now called *Gabriel's horn* [also called Torricelli's trumpet], is not intuitive. Torricelli was amazed at his discovery, saying, "It may seem incredible that although this solid has an infinite length, nevertheless none of the cylindrical surfaces we considered has an infinite length but all of them are finite." For those familiar with integral calculus, this is a rather straightforward problem, but considering the time Torricelli proved it, predating calculus, it is quite remarkable. The finite volume is given in modern notation by the improper integral

$$\int_1^\infty (\pi/x^2)\, dx = \lim_{a \to \infty} \int_1^a (\pi/x^2)\, dx = \lim \left[- \pi/x\right]_1^a = \lim \left[- \pi/a + \pi\right] = \pi$$

Torricelli's proof is much different and more complex. At the time this result generated what appeared to be a paradox, namely: while it seems it would take an infinite amount of paint to coat the outside surface of Gabriel's horn, but as its volume is finite, the interior surface could be coated with a finite amount of paint. The paradox is resolved by considering its assumptions and noting that much of the interior of the horn would be inaccessible, because paint molecules have thickness, and even if they did not, it would take an infinite amount of time for the paint to run down the infinite length of the horn.

Figure 10. 6

Torricelli prepared a treatise that extended Galileo's work on the parabolic motion of projectiles and also wrote on the motion of fluids, which was extremely important to the early development of hydrodynamics. *Opera geometrica* (1644, *Geometric Works*) was his only book published during his lifetime. Some of his manuscripts have been published since his death, while others remain to be translated. When he died, probably of typhoid fever at the age of 39, he was buried in a common grave.

Quotation of the Day: “The Geometer has the special privilege to carry out, by abstraction, all constructions by means of the intellect. Who, then, would wish to prevent me from freely considering figures hanging on a balance imagined to be an infinite distance beyond the confines of the world?” – Evangelista Torricelli