René Thom

French differential topologist **René Frederic Thom** (September 2, 1923 – October 25, 2002) earned international recognition for his work in topology. With Hassler Whitney he co-founded the two new mathematical fields of differential topology and singularity theory. During the 1950s Thom created cobordism theory and classified generic singularities of smooth maps. Cobordism is a means of classifying manifolds (higher dimensional analogues of spheres and other curved surfaces). For this work Thom was awarded the Fields Medal in 1958. He is most widely known for his development of catastrophe theory, a calculus of discontinuities, which emerged with the publication of his *Stabilité structurelle morphogenèse* (Structural Stability and Morphogenesis, 1972). Morphogenesis is the structural change occurring during the development of an organism. In simple terms, catastrophe theory is the mathematics of sudden change. More rigorously it is a branch of topology that is concerned with the way in which nonlinear interactions within systems can produce sudden and dramatic effects. Catastrophe theory is meant to provide an understanding of the nature of sudden change with the same mathematical rigor that is used to understand gradual change.

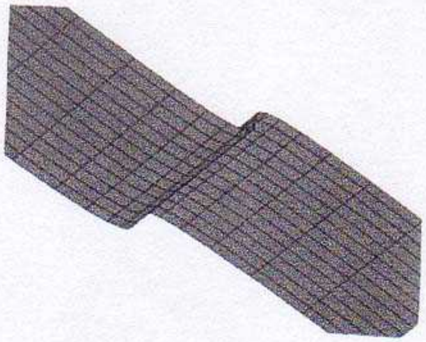
Thom was born in Montbéliard, near the Swiss border, where his parents were shopkeepers. He attended the local primary school (at which he reputedly learned to visualize in four dimensions) and the Collège Curvier. He received a baccalaureate in elementary mathematics from Besançon in 1940 just before the Nazi invasion of France. His parents sent him and his brother south to protect them from the conflict, and after a few months he continued his education in Lyons, completing a baccalaureate in philosophy from Lyons University in 1941. His first attempt to be admitted to the École Normale Supérieure in Paris was unsuccessful, but in 1943 he was accepted. Thom was greatly influenced by Henri Cartan and the Bourbaki approach to mathematics. Before he had finished his studies WWII ended and Thom followed Cartan to Strasbourg, where he earned his doctorate for a 1951 thesis "Fibre Spaces in Spheres and Steenrod Squares" directed by Cartan. It contained the foundations of his theory of cobordism. A fellowship allowed him to travel to Princeton University in the United States in 1951 after which he returned to France to teach at Grenoble (1953-54) and then at Strasbourg (1954-63). In 1964 he moved to the Institut des Hautes Études Scientifiques at Bures-sur-Yvette. At this time, the star of the Institut was Alexander Grothendieck, who apparently treated Thom very badly. At his retirement conference in 1988 the still bitter Thom remarked: "I have never understood anything that Alexander Grothendieck has done, and I haven't the least desire to do so."

Thom moved from research in strictly mathematical areas to explore more general notions and to seek mathematical ways of describing and predicting phenomena in the "soft" sciences. He admitted this was in part because he was in the shadow of Grothendieck, and had nothing new to offer. Mathematical applications flourish in physics and engineering, because of the useful tool of the differential calculus, but this same tool is less useful in the biological, behavioral and social sciences. For instance, in psychiatry, calculus can't be employed to predict when a psychotic individual is likely to snap and do harm to himself or others. How does one predict when an earthquake will occur or a bridge collapse? How does one expect the unexpected like a stock market crash, a prison riot, or the outbreak of war? These are instances of phenomena of unanticipated changes. Mathematically it's like moving smoothly along a continuous path and suddenly encountering an abrupt (and seemingly unpredictable) break in the path. Thom was the spiritual leader in the development of catastrophe theory, used to study and classify phenomena characterized by sudden shifts in behavior arising from small perturbations, that is, a mathematical treatment of continuous action producing a discontinuous result.

The mathematical theory classifies ways in which critical points (maxima, minima, saddles) can coalesce as parameters are altered. Thom used the theory to describe the change of a system's structure along a continuous "morphogenetic landscape" including occasional jumps. With it he attempted to explain how discrete forms can emerge out of homogeneity, and how continuous causes can give rise to discontinuous effects. At the heart of the theory is that big changes in behavior can be brought about by small changes in circumstances. The name of the theory was suggested because normally if a large, bad, unexpected discontinuity happens it is liable to be called a catastrophe. However, there are many other examples of sudden, unexpected change not nearly as dire as the name "catastrophe" suggests, such as reflection or refraction of light through water, waves breaking on the shore, or even water boiling.

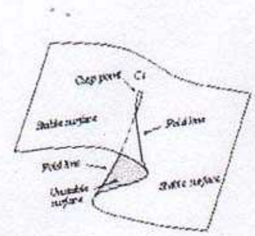
Though there are an infinite number of ways for a system to change continuously, there are only seven structurally stable ways for such a system to change discontinuously. Thom proposed seven qualitatively different types of elementary catastrophes, which are sufficient to describe situations involving three dimensions of space and one of time. He did not give a complete proof of the classification, but outlined the steps required to carry out the proof, which was completed by Bernard Malgrange. The seven catastrophes are: the *fold*, the *cusp,* the *butterfly,* the *swallowtail*, the *hyperbolic umbilic*, the *elliptic umbilic*, and the *parabolic umbilic*. The first four are named for the shapes suggested by their curves and the last three are reminiscent of umbilical tethers classified according to the basic conic sections [Figures 9.1 – 9.7].

Elementary Catastrophes



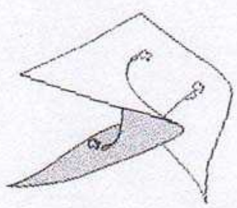
Fold

Figure 9.1



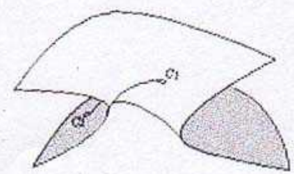
Cusp

Figure 9.2



Swallowtail

Figure 9.3



Butterfly

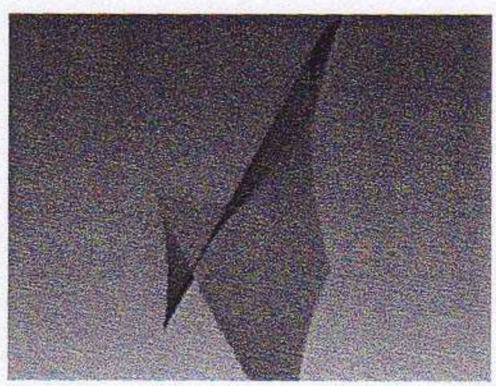
Figure 9.4

Hyperbolic Umbilic

Figure 9.5

Elliptic Umbilic

Figure 9.6



Parabolic Umbilic

Figure 9.7

Although Thom introduced the theory in the late 1960's it didn't appear in book form until 1972 with the publication of Christopher Zeeman, who named the theory, brought it to the attention of the general public and pioneered its application to the biological and behavioral sciences. He invented a Catastrophe machine, a mechanical device to illustrate how a small perturbation can give rise to a discontinuous consequence. Numerous applied mathematicians claimed that they had been using these ideas for years. Catastrophe theory was fashionable in the 1970's and 1980's but has fallen from favor, possibly because those for whom it was intended - biologists, behavioral scientists and social scientists – didn't have the necessary mathematical background. The motivation for its development was laudable since the need to develop appropriate mathematical tools for use in the biological, behavioral and social sciences is not disputed. Some critics claimed catastrophe theory was little more than an intellectual fad. Thom reluctantly announced its demise in 1990: "Catastrophe theory is dead. For as soon as it became clear that the theory did not permit quantitative prediction, all good minds … decided in was of no value." But Ian Stewart in *Life's Other Secret* (1998) wrote: "…it completely revolutionized bifurcation theory. It was a bloodless revolution, accomplished under an assumed name (singularity theory), and it took place largely within mathematics, so hardly anybody noticed." Catastrophe theory paved the way for the more influential chaos theory.

In addition the Fields Medal, Thom was awarded the Grand Prix Scientifique de la Ville de Paris in 1974 and made an honorary member of the London Mathematical Society in 1990. In 1983 Salvador Dali painted *Topological Abduction of Europe: Homage to René Thom*. It is an aerial view of a seismically fractured landscape juxtaposed with the equation that strives to explain it.

Thom was described as a shy, reserved man, with a gentle wit and a great skepticism, often amused by the human condition. He liked to gently prod his colleagues into questioning their assumptions. It was claimed that what he did not understand, he did not like, and referred to these things disparagingly as "Anglo-Saxon." He died on October 25, 2002, survived by his wife Suzanne, whom he had married in 1951, and three children.

Quotation of the Day: "When so many scholars in the world are calculating, is it not right that some, who can, dream?" – René Thom