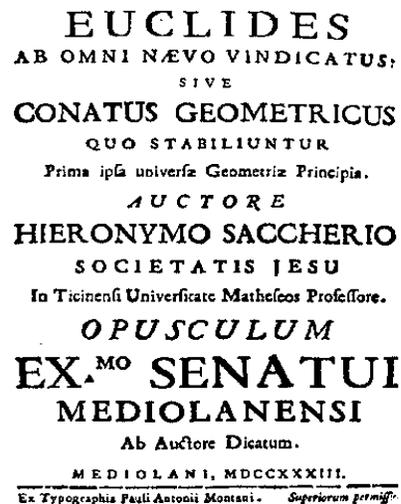


Giovanni Girolamo Saccheri

Italian Jesuit and logician **Giovanni Girolamo Saccheri** (September 5, 1667 – October 25, 1733) made an elaborate and valiant effort to prove the parallel postulate of Euclid. Unfortunately, he was so thoroughly convinced that Euclid's was the only valid geometry that he missed the chance to discover non-Euclidean geometry. It may be that Saccheri concluded that at the time, it was more prudent not to question the infallibility of Euclidean geometry. He was born in San Remo, Genoa and died in Milan. He entered the Jesuit Order at Genoa in 1685 and was sent to Milan in 1690, where he studied philosophy and theology at the Jesuit College, earning a doctorate of theology. While there he was encouraged to take up the study of mathematics by his teacher Tommaso Ceva. Saccheri was ordained in 1694 at Como. In the following years Saccheri taught at various Jesuit Colleges including Turin (1694 – 1697). He held the chair of mathematics at Pavia from 1697 until his death.



One could argue persuasively (and several have) that the outstanding problem of Western mathematics for two millennia before Saccheri's time was that of "Euclid's fifth postulate." There were many flaws in Euclid's *Elements* as he attempted to establish geometry as a deductive system. The most serious of these was that his postulates were insufficient to establish the geometry he was building. This in no way diminishes the genius of Euclid, as these assumptions were so subtle that no one recognized them as such until after the development of non-Euclidean geometries in the 19th century. But almost from the time the ink was dry on Euclid's papyrus, mathematicians (possibly even Euclid himself) began to raise questions about the status of the last of the five fundamental postulates on which his geometry was built. In modern language, the postulate states: "If two straight lines in a plane are cut by a transversal making the sum of the measures of two interior angles on the same side of the transversal less than 180° , then the two straight lines will meet on that side of the transversal." This assertion is linguistically much more complex than the other four; it does not seem to share their straightforward status as a "self-evident" property of geometry – Euclid's own standard for a postulate. In form and "feel" it is much more akin to Euclid's theorems than it is to the other postulates. But, if that is so, then there must be a proof that logically derives this statement from the other postulates. This was the goal that Saccheri set for himself, one that others had sought before, but in

vain.

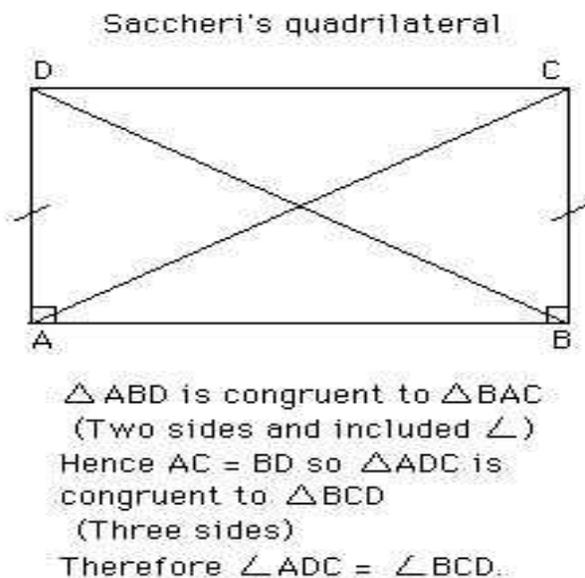
Saccheri had studied the “proofs” of the parallel postulate concocted by Nasîr-Eddin, the 13th century Persian editor of Euclid and that of John Wallis, who, in 1663, published a criticism of Nasîr-Eddin. Saccheri’s method was to develop the consequences of denying the parallel postulate while retaining the other postulates. He expected that in doing so he would develop a geometry that was self-contradictory, as he was certain the parallel postulate was a necessary truth. However, despite heroic efforts Saccheri never succeeded in arriving at a contradiction. What he did accomplish without realizing it was to lay the foundations for the first non-Euclidean geometry. He would be remembered with greater respect had he not lacked a certain intellectual integrity. Rather than recognize that he had failed to find a contradiction because none could exist and that the Euclidean axioms together with the denial of the parallel postulate were the foundations of a new geometry, he published his *Euclides ab Omni Naevo Vindicatus* (*Euclid Vindicated from All Faults*, 1773). It showed him to be an insightful and imaginative geometer, presaging much of what Gauss, Bolyai and Lobachevsky would develop 160 years later; but in the end he twisted his reasoning to such a point that he became convinced that he had succeeded in finding a contradiction. In fact, had he thus succeeded, he would not have “vindicated” Euclid; he would have exposed a structural flaw in Euclid’s postulating what was a provable theorem. However, from some insight of which we have no knowledge, Euclid rightly realized that he needed the postulate. Saccheri’s book was all but forgotten until Eugenio Beltrami rescued it from oblivion in 1889.

Given a line segment AB, construct segments AC and BD on the same side of AB such that AC = BD and both AC and BD are perpendicular to AB (Figure 9. 9). Then join C and D forming what Saccheri prosaically dubbed a *semi-equilateral bi-rectangular quadrilateral*, but known today as a *Saccheri quadrilatera*

Figure 9.9

Without using the Parallel Postulate, Saccheri was able to prove that the angles ACD and BDC were congruent. He called these two angles *summit angles* and observed that one and only one of the following statements is true.

1. *Right Angle Hypothesis*: The summit angles are right angles.



2. *Obtuse Angle Hypothesis*: The summit angles are obtuse angles.
3. *Acute Angle Hypothesis*: The summit angles are acute angles.

Saccheri was able to show that the Parallel Postulate was a consequence of the *Right Angle Hypothesis*. He planned to show this hypothesis held by eliminating the *Obtuse* and *Acute Hypothesis*. He showed that the *Obtuse Angle Hypothesis* contradicted the infinite length of a line. Saccheri then set about the task of obtaining a contradiction by assuming the *Acute Angle Hypothesis* but despite his success in misleading himself he never reached such a contradiction. In fairness to Saccheri, it is important to note that the logical and mathematical reasoning methods found in his *Euclides* and his other important work *Logica Demonstrativa* (1697) have become part of mathematical logic.

Quotation of the Day: “During the long period from the time of Euclid to the nineteenth century, many professional and amateur mathematicians ... tried to deduce the disquieting Fifth Postulate from the others by the “indirect method.” They hoped that its denial would lead to a contradiction. The most prominent of these pioneers was the Jesuit Girolamo Saccheri, who discovered, for the sole purpose of demolishing them, many theorems of what we now call hyperbolic geometry.” – H.S.M. Coxeter