George Pólya

The undisputed father of mathematical problem solving is **George Pólya** (December 13, 1887 – September 7, 1985), one of the giants of classical analysis in the 20th century. He considered solving problems to be a practical art, one that can be taught and learned. His books on the subject *How To Solve It* (1945) and the two-volume set *Mathematics and Plausible Reasoning* (1954*)*, and *Mathematical Discovery* (1962) are classics. The first, which has been translated into 21 languages, has sold more than a million copies over the years. Pólya's influence goes far beyond the number of his books that were sold. His books form the basis for enlightened thinking in mathematics education in which student understanding is the goal rather than student memorization. Teachers throughout the world have enthusiastically adopted his suggestions and found that they worked. No one can estimate the number of students who, empowered with Pólya's methods, have found that they actually can understand what they are doing in solving problems. However, the crusade to have Pólya's ideas employed has not converted all the "heathens"; there are still strong factions who stress "learning how to do" over all other mathematical virtues.

Pólya was born in Budapest, Hungary and died in Palo Alto, California, almost 98 years later. Both of his parents were born Jewish but converted to Catholicism. His father was born Jakab Pollák, a surname suggesting Polish origins. Jakab changed his name to the more Hungarian , believing this would help him obtain his goal of a university position. He was a talented solicitor, but because he often accepted cases without fees, he was not a financial success. George, who was originally called György, attended Dániel Berzsenyi Gymnasium, where he earned a fine academic reputation, but did not shine in mathematics. Initially he resisted the career that fate had in store for him, because as he later recalled his mathematics instructors who should have been his models were “despicable teachers.”

Even at an early age George had great skill for analyzing and solving problems. His uncle encouraged him to pursue a mathematical career but Pólya wanted to become a lawyer like his father. He entered the University of Budapest, but became bored with all the legal technicalities he was required to memorize. After reading Charles Darwin’s *The Descent of Man*, Pólya briefly took up the study of biology, but when his brother insisted there was no money to be made in the subject, George shifted to languages and literature. Next he turned to philosophy but to better understand it he had to learn mathematics and he was hooked. He was awarded a PhD in mathematics from the University of Budapest (1912) for an essentially unsupervised thesis in geometric probability. He spent the following year in Göttingen.

Pólya’s first job was tutoring the son of a baron. His pupil struggled with mathematics because he lacked problem-solving skills. To deal with this Pólya began developing his method of problem solving, which he hoped would not only work for his student but for others facing a similar challenge. He was convinced that problem solving was not some special ability that some are born with and others not, but rather was a practical skill that could be taught to anyone, and if students are to have a chance of understanding mathematics, it must be learned. In 1914 he was invited to teach at Zurich and while in Switzerland he made two major discoveries. One was Stella Weber, with whom he spent 67 years of married life. The second discovery came to him as he took walks in a local park. It led to what he called the “random walk problem.” Some years later he published a paper proving that if one continued a walk on a grid long enough, one was certain to return to the starting point. In 1921 he investigated what he called “street networks,” which are now referred to as “lattices.”

In 1924 Pólya was the first International Rockefeller Fellow, spending a year in England, where he worked with G.H. Hardy and John Littlewood. Nine years later, he was once again a Rockefeller Fellow, spending the year at Princeton University. Although his main mathematical interest was in real and complex analysis, he also made contributions to probability, combinatorics, algebra, number theory, voting systems and astronomy. Other mathematicians' elaborations on his major contributions have become the foundations of several important branches of mathematics. Independently, Pólya and Hilbert conjectured that the zeros of the Riemann zeta function correspond to the eigenvalues of a self-adjoint Hermitian operator. His main contribution to combinatorics is the enumeration theorem. He collaborated with Hardy and Littlewood on the first systematic study of inequalities.

In 1940 George and Stella moved to the United States because of their concerns about Hitler and the Nazis in Germany. He taught at Brown University for two years, and then spent a short time at Smith College before finally moving to Stanford in California, in 1942, where he stayed for the rest of his life. He retired in 1954 but continued to teach until 1978. Pólya was a masterful storyteller, a man of rare wit, insight, enthusiasm, and tremendous curiosity. He was a genuinely friendly individual who enjoyed entertaining visitors by showing them pictures of famous mathematicians he had known, and recalling delightful and amusing instances in their lives. While Pólya was correct in believing that he could teach others his skill for problem solving, it is a shame that his other strengths can't be taught as easily. In "George (1887-1985)," *Mathematics Magazine*, December 1987 M.M. Schiffer stated: "The driving force in his research was the search for beauty and the joy of discovery."

Before leaving Europe, wrote a German draft of his book, *How To Solve It*. Remarkably, three U.S. publishers passed on the project before the Princeton University Press agreed to publish the English version in 1945. In it into modern usage a *heuristic* method for solving a problem. The term , which dates to the early Greek philosophers, refers to a useful method that doesn't work every time. He wrote, “Heuristic reasoning is reasoning not regarded as final and strict but as provisional and plausible only, whose purpose is to discover the solution of the present problem.”

complained that textbooks presented mathematics as a rigorously organized set of axioms followed by proofs of theorems derived from them. This gives students the wrong impression of how mathematics is actually done. When trying to solve a problem, develop a theory or prove a theorem, mathematicians try one approach after another, make numerous false starts, which may lead them down blind alleys, have sudden flashes of intuition, lucky guesses, and so forth. Mathematicians have their personal rules of thumb, or heuristics, for doing their work. In his book described the steps and strategies that he observed successful mathematicians using as they went about solving problems. did not invent these methods, but made them widely available to students, suggesting a framework for teaching meta-reasoning in mathematical problem solving.

Pólya’s strategy for solving problems consists of principles so obvious one might wonder why no one had enunciated them before. Well, perhaps others had, but Pólya taught another educational truism that cannot be expressed too often. To be able to solve problems, one must ask “good” questions. Within his elaboration on his principles Pólya suggested some of these “good” questions. Pólya’s four basic principles are 1. Understanding the problem, 2. Devising a plan, 3. Carrying out the plan, and 4. Looking back. The principles and strategies that set forth on problem solving and plausible reasoning need not be limited to mathematics. His general heuristics provides strategies for solving all kinds of problems.

Pólya’s classical books are a must read for teachers who hope to instruct their students in strategies not only for solving problems but also for formulating problems to be solved. He recognized that if his methods were to be successfully shared with students, their teachers had to amend the way they presented mathematical material and to alter their role in the learning process. His *Ten Commandments for Teachers*, found in his book *Mathematical Discovery* (1981) are as follows:

1. Be interested in your subject.
2. Know your subject.
3. Know about the ways of learning: the best way to learn anything is to discover it by yourself.
4. Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place.
5. Give them not only information, but "know-how," attitudes of mind, the habit of methodical work.
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving the problems to come – try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once – let the students guess before you tell it – let them find out for themselves as much as feasible.
10. Suggest it; do not force it down their throats."

Not everyone agrees on what exactly is a problem. There may not be a perfect definition of "problem," but it is possible to make a distinction between a problem and a task. With a task, one knows what to do and how to do it; with a problem, neither what to do nor how to do it may be known, at least initially. One of the first steps in problem solving is to carefully describe the problem one wishes to solve, and this may not be easy. Whereas in classrooms, problems are presented to students to solve, in the real world, problems are not ordinarily assigned by someone to be solved, but are discovered to need solving. Even some of those successful in "solving" problems, may not understand problems. Many have been taught to mimic a procedure, and as good imitators they get correct answers, but may have little awareness of what they have done or of the value of their labor.

If students merely imitate procedures, no plan is needed, and this becomes obvious when they are asked to work on a new, unfamiliar problem. They do not think about the problem or a plan of attack. Instead they ripple through their Rolodex of memorized techniques, hoping to find a trick that works. For them, carrying out a plan consists of mindlessly imitating the technique they have learned. After they get an answer, they don't look back, not even to see if their result makes any sense in the context of the problem.

As the primary goal of good students is to understand things, it should be the goal of good teachers to help others understand. This cannot be accomplished through memorization or imitation. By the time some people become acquainted with Pólya's *How To Solve It*, they have already developed their own strategies for learning mathematics. There seems to be an extra principle that should be added to Pólya's – one preceding number four. That is, pretending you are *interested* in finding the solution of a problem until you get to the point that you are. Teaching mathematics is difficult for reasons that appear to be unique to the subject. One of these is that there are several languages integral to the learning process with which teachers and pupils alike must be conversant.

There is the mathematics itself, the language of mathematics, the language of logic, and a *meta-language*, which teachers use to instruct students in the language of mathematics so that mathematics itself may be understood. For instance, meta-language is used to provide students with an intuitive understanding of mathematical concepts before they are formalized or abstracted. By examining meta-languages used by teachers it is possible to determine likely communication failures that prevent willing instructors from helping willing students realize their mathematical potential in problem solving as well as developing proficiency with mathematical techniques. Teachers have their own peculiar meta-languages, formed by their personality and experiences. A search of scholarly literature reveals that this is essentially an under explored area of educational research, particularly as applied to mathematics. It is necessary to narrow the gap between teachers' expectations and students' perception and acceptance of them.

Quotation of the Day: "A mathematics teacher is a midwife to ideas." – George Pólya