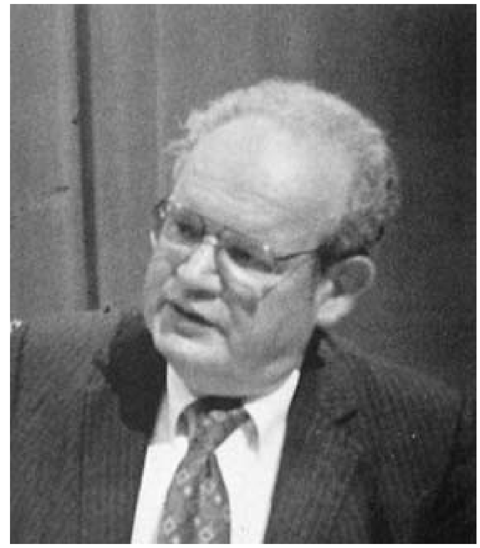
Benoit Mandelbrot



Polish-born mathematician **Benoit Mandelbrot** (November 20, 1924 -) is expert in an area where mathematics, science and art merge. He has a visual mind, which allows him to solve problems with great leaps of geometric intuition. Having no formal training in algebra, he once passed an important exam with the highest grade by mentally translating all of the problems into pictures. Concentrating his research on extreme and unpredictable irregularity in natural phenomena in the physical, social, and biological sciences, he invented a basis for the theoretical investigation of complex dynamics, finding order in apparently erratic shapes and processes. His study of processes with unusual statistical properties, and of everyday forms of nature, such as mountains, clouds and the path traveled by lightning, made him one of the pioneers of Chaos Theory or Dynamical Systems. He concluded that the almost totally smooth patterns used in mathematical physics to describe nature were flawed and incomplete. He expressed the thought: "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line." He went beyond Einstein's theories of the fourth dimension by establishing that besides including the first three dimensions, there also are gaps or intervals between them that are fractional dimensions.

Mandelbrot was born into a Lithuanian-Jewish family in Warsaw. When he was eleven the family moved to Paris, where his uncle Szolem Mandelbrojt, who had succeeded Jacques Hadamard as Professor of Mathematics at the Collége de France, took responsibility for his nephew's education. A founding member of the innovative group of French mathematicians "Nicholas Bourbaki," Szolem somehow turned young Mandelbrot against pure mathematics. Benoit attended the Lycée Rolin in Paris up until the beginning of WWII when the family moved to Tulle in south central France. For two years, Mandelbrot and his younger brother wandered from place to place, mindful that due to their Jewish heritage they were in danger from the Germans occupying most of France. He entered the École Normale in Paris, but as the curriculum was pure mathematics, he left after only one day. In 1944 he enrolled at the École Polytechnique. After graduating in 1946, he received a scholarship to study at the California Institute of Technology. Three years later he returned to France with a master's degree in aeronautics, and then spent a year in the French Air Force.

Looking for a Ph.D. topic at the University of Paris, Mandelbrot came across *Human Behavior and the Principle of Learned Effort* by George Zipf, in which the author discussed examples of frequency distributions in the social sciences that did not follow the normal distribution. The first half of Mandelbrot's 1952 Ph.D. thesis was on Zipf's claims about word frequencies and the second about statistical thermodynamics. John von Neumann invited Mandelbrot to the Institute for Advanced Study at Princeton, where he spent the academic year 1953-54. He returned to Paris to become an associate at the Institut Henri Poincaré, and in 1955 married biologist Ailette Kagan. He taught at the University of Geneva (1955-57) and then became a junior professor of applied mathematics at Lille University (1957-58), while also teaching mathematical analysis at the École Polytechnique. Not happy with the state of French mathematics at the time, and his interests lying in "the more exotic forms of statistical mechanics and mathematical linguistics," Mandelbrot felt a mathematical outcast. His interest in mathematics was not with its flowering but instead as a means of describing the previously undescribed. He responded to the challenge of finding better mathematical models of physical objects and processes that are all around us. His genius is in seeing the links between dynamical systems and finding ways to describe them.

In 1958 Mandelbrot became a member of the research staff at the IBM Research Center in Yorktown Heights, New York. There he worked on mathematical linguistics, game theory, and economics before being asked to investigate the problems of noise on telephone wires used for computer communications. He found that the noise was random in timing, occurring in bursts on ever shorter time scales, the distribution of the noise spikes always remaining a scaled-down version of the whole. He used the IBM computers to explore other dynamical systems, those like the stock market, ecosystems, the weather, the human body, etc., that are in constant flux. To better understand these systems, he needed mathematical models, but the traditional ones that served so well in physics and astronomy gave an incomplete picture of their behavior.

Chaos theory is not the study of disorder, but rather it is the science of complexity. Chaos is produced spontaneously, and in a chaotic system there are instances in which the mix of order assumes new directions. These concentrations are called bifurcation points, that is, where the system branches off. Pioneers in chaos theory discovered that graphing non-linear equations using feedback loops allowed them to look at pictures of these systems. Mandelbrot conceived of what he called "The Noah Effect," which suggests that change occurs in discrete jumps. He concluded that the variations in stock market prices, the probabilities of words in English, and the fluctuations in turbulent fluids, might be modeled by unusual statistical properties. He studied the geometric features of these processes and concluded that a unifying feature was their self-similarity, that is, they looked or behaved the same when examined from far away or nearby. In the mid-1970's, Mandelbrot coined the word "fractal" for these underlying shapes or behaviors that had similar properties at a discrete set of magnifications. The name is derived from the Latin *fractus*; the verb *frangere* means, "to break," "to create irregular patterns." Using computers he found that many shapes in nature – even those of ferns and the holes in Swiss cheese - could be described and replicated using fractal formulas. In the process he devised a geometry with fractional dimensions.

In the 1970s Mandelbrot revisited a paper by Gaston Julia given him by his uncle in 1945, which the older man thought was a potential source of interesting problems. Julia worked with mathematical expressions involving complex numbers. He fed a number into an equation and fed the calculated answer back into the original equation, repeating this process over and over. Mandelbrot was able to use computers to speed up the process. As a result he created what is known as the "Mandelbrot Set," [Figure 11.9] formed by picking a point z_0 in the complex plane, and calculating as follows: $z_1 = z_0^2 + z_0$; $z_2 = z_1^2 + z_0$; $z_3 = z_2^2 + z_0$; … If the sequence z_0, z_1, z_2, z_3, … remains within a distance of 2 of the origin forever, then the point z_0 is said to be in the Mandelbrot set. If the sequence diverges from the origin, then the point is not in the set. Mandelbrot introduced his concept of *fractals* and his set in an invited address given in 1973 at the Collége de France. The lecture was published in expanded form as *Les Ojects Fractals: Forme, Hasard et Dimension* (1975), and a revised and expanded English edition *The Fractal Geometry of Nature* (1982). Some features of the Mandelbrot set include: the area is unknown but it is quite small; the length of its border is infinite; the shapes along the ever increasing border occur an infinite number of times, rotated, distorted and shrunken; all the black areas are connected; the set can be used as a very inefficient means of calculating π; and it has infinite detail – it is possible to zoom in forever.

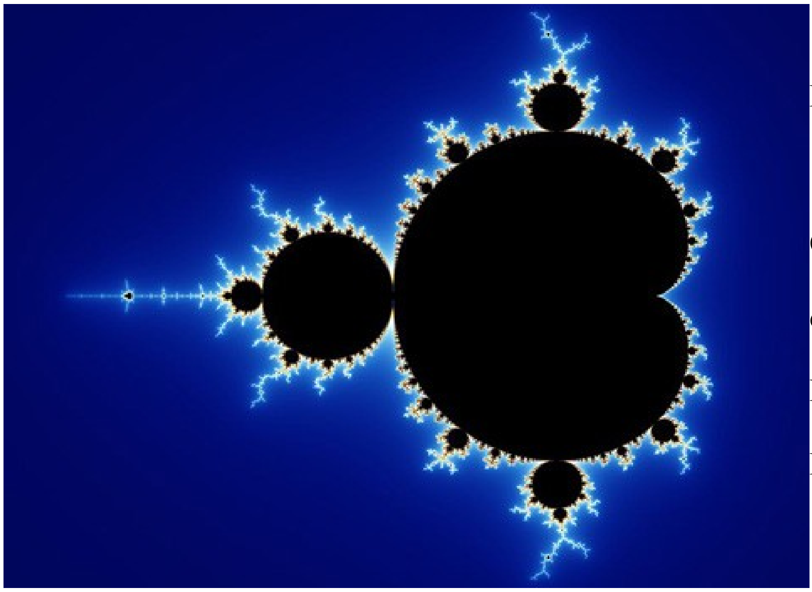


Figure 11.9

Objects now known as fractals were discovered and explored long before Mandelbrot coined the term. In 1872 Karl Weierstrass gave an example of a function that was everywhere continuous but nowhere differential. The graph of this function would now be called a fractal. In 1904 Helge von Koch gave a geometric definition of function, which is known as the Koch snowflake and very definitely is a fractal. Before the advent of the term "fractal," geometric objects whose self-similar structure occurred at different levels of magnification were commonly known as *monster curves*. In an attempt to understand geometric objects with unusual properties, Constantin Carathéodory and Felix Hausdorff generalized the concept of dimension to include non-integer values. Abram Samoilovitch Besicovith extended the so-called Hausdorff dimension to highly irregular sets. For this reason the Hausdorff dimension is sometimes referred to as the Hausdorff-Besicovitch dimension. It is less frequently called the capacity dimension or fractal dimension. Mandelbrot says, "A fractal is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension." The topological dimension of a smooth curve is one; of a sphere is two, and a solid of three. Luitzen Brouwer gave the formal definition of topological dimension in 1913. The Hausdorff-Besicovitch dimension is a measure of how "complicated" a self-similar figure is. In a rough sense, it measures "how many points" lie in a given set. A plane is larger than a line, while the dimension of the Sierpinski triangle is somewhere in between these two sets.

Initially fractals and fractal geometry were considered merely a mathematical curiosity, but they have increasingly provided insights into natural phenomena such as earthquakes, and have found application in many areas of human activity such as polymers, nuclear reactor safety and economics. It is used to describe irregularly shaped objects or spatial non-uniform phenomena that Euclidean geometry and its perfect squares, pyramids, spheres, etc. cannot. Computer graphics has been one of the earliest applications of fractals. Computer games and cinema special effects owe their remarkable realistic images to the study of. Fractal landscapes first appeared in the films *Return of the Jedi*, *Star Trek II*, and *The Last Starfighter.*

Mandelbrot taught economics at Harvard, engineering at Yale, and physiology at the Einstein College of Medicine. In 1985 he received the Barnard Medal for Meritorious Service to Science, awarded every five years by the National Academy of Sciences, and in 1986, the Franklin Medal for his development of fractal geometry. In 1987, he was appointed Abraham Robinson Professor of Mathematical Sciences at Yale. He officially retired from IBM in 1993, but continues to work at Yale and at IBM as a fellow emeritus.

Quotation of the Day: “Fractal geometry reveals that some of the most austerely formal chapters of mathematics had a hidden face: a world of beauty unsuspected until now.” – Benoit Mandelbrot