Mark Kac

Polish-born U.S. mathematician **Mark Kac** (August 3, 1914 – October 26, 1984) pioneered in the mathematical theory of stochastic processes, particularly in its applications to statistical physics. He was the first to study the distribution of the absolute area of Brownian motion using what has since been known as the "Feynman-Kac formula, named for him and Richard Feynman" Brownian motion describes the molecular movement of liquids, which moves small particles suspended in it. Kac treated this work in his book *Probability and Related Methods in the Physical Sciences* (1959). He is also known for his works in collaboration with Paul Erdős in "probabilistic number theory." In his lecture notes *Integration in Function Spaces and Some of Its Applications*, Kac asserted "Mathematicians, or rather mathematical analysts, are divided roughly speaking into two classes: the "calculators," that is, those who look for exact formulas, and the "estimators," that is, those who live by inequalities. I belong to the first class."

Kac was born to a Jewish family in Krzemieniec, then part of Russia, later in Poland, and now in Ukraine. According to the Gregorian calendar, used by most of the world in 1914, he was born on August 16, but his birth certificate lists the date as August 3. At the time, Poland was part of the Russian Empire, which had not switched from the Julian calendar. During WWI, Kac's family was evacuated further east in Russia, where his father supported his family by tutoring in their one room apartment. Fascinated by the mathematical lessons he heard in his home, Mark convinced his father to teach him mathematics. Mark discovered a new derivation of Cardano's formula for the solution of the cubic equation, an accomplishment that cost his father five Polish zlotys as a prize. The family returned to Poland in 1921 where a governess taught Kac French, Russian and some Hebrew, but not Polish. At eleven he entered the Lycée of Krzemieniec, where he finally learned his native language. After graduation, Kac entered the Jan Kasimir University of Lvov, where Hugo Steinhaus taught him mathematics and introduced him to probability. After obtaining his first degree, he remained at the university, earning a doctorate in 1937.

When Hitler came to power, Kac had a premonition of how bad things would become in Europe, so he decided to leave Poland. He first sought an academic position in England, but when this didn't pan out he turned his sights to the United States. He finally was given a six-month visitor's visa with the requirement that he buy a return ticket to guarantee he would go back to Poland after the half-year. When Germany attacked Poland in 1939, Kac was safely in the U.S. at Johns Hopkins unable to return home. His family was not so fortunate. His parents and his brother and many of his friends perished in the mass executions in Krzemieniec (1942-43). Kac secured a position as a mathematics instructor at Cornell University. He became a U.S. citizen in 1943 and was promoted to full professor in 1947. He left Cornell in 1961 to join the faculty at Rockefeller University in New York City, and twenty years later, he moved to the University of Southern California where he spent the remainder of his career.

Among Kac's most intriguing questions was "Can one hear the shape of a drum?" It was known that the shape of a drum determines sound, that is, one can "hear" a drum's area and perimeter. But consider the converse; does sound determine the shape of a drum? In other words, if you close your eyes and listen to differently shaped drums being played, would you be capable of distinguishing the shape by the sound or vibration frequencies that you hear? The interior of a drum vibrates when it is struck while the boundary frame remains rigid. Thus, Kac's question can be restated as, "Can drums exist with distinct geometric shapes that vibrate at the same characteristic frequencies?" A mathematical drum is not the usual musical instrument. It is any shape in the plane that has an interior and a boundary. A mathematical process is determined that approximates the way an ideal note is produced. In the mathematical model, each tap on a drum contains an infinite list of pitches that go arbitrarily high. Then the mathematical question is: Are there two drums with different shapes which give rise to the same infinite list of pitches?

In 1991 Carolyn Gordon and David Webb then both at Washington University in St. Louis, and Scott Wolpert of the University of Maryland proved a theorem that answered the previous question positively, thus negatively answering Kac's question. Using methods from spectral geometry and group theory, Gordon and Webb came up with two multisided polygons (Figure 8.1) with equal areas and perimeters but different geometric shapes. In principle, when thought of as drums, they would sound exactly alike, both generating the same normal-mode frequencies.

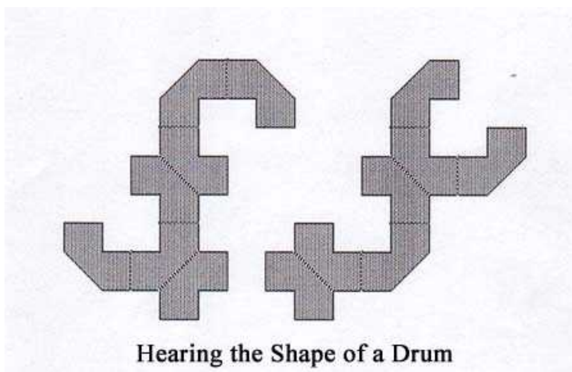

Figure 8.1

Physicist Srinivasa Sridhar and his colleagues at Northeastern University in Boston demonstrated the theorem by constructing the two "drums," fabricated by Gordon and Webb. They consisted of two cavities, each corresponding to one of the pair of shapes. They were constructed out of copper and had eight flat sides. Microwaves were sent through tiny openings in the "drums" and their strength measured over a range of frequencies at another location. As a result Sridhar was able to establish the frequencies of the normal modes of each cavity. Sure enough, they were practically identical, the discrepancies being due to the imperfections in the drum's assembly.

Professor Peter Perry of the University of Kentucky offers an answer to the question why should anyone be interested in Kac's query:

> "Kac's question means: does this infinite sequence identify the drum's shape uniquely, as a fingerprint identifies a human being? The answer to the question is important because analogous questions occur in applied fields such as medical imaging and seismology. In medical imaging, an MRI scan measures the response of a patient to magnetic fields; from this data, an image of the patient's body is constructed. In the geophysical arena, detectors on the surface of the earth measure the seismic waves produced by a controlled explosion, and from this data the shape of the Earth below the surface can be reconstructed, and oil or mineral deposits located. By better understanding the mathematical model in Kac's problem, the mathematical foundations of medical imaging and seismology might be better understood."

Kac's *Enigmas of Chance* (1985) is one of the finest scientific autobiographies ever written. His great charm, so evident in his speech, enriches his writing of his love of life and love of work. Among the gems is "The mere truth of a proposition is not sufficient to establish it as part of mathematics. One looks for 'usefulness,' for 'interest,' and also for 'beauty.'" His views on proof are summarized in the statement: "A proof is what convinces a reasonable man; a rigorous proof is that which convinces an unreasonable man." In a chilling observation, Kac wrote "As I look back on my life I marvel at the improbable assortment of people who, independently of each other, cooperated to keep me from being incinerated in the ovens of Auschwitz or Belsen…" In 1939 Kac married Katherine Mayberry, with whom he had a son and a daughter. After his death at the age of seventy, the Mark Kac Memorial Fund was set up to support memberships in the American Physical Society and sponsor subscriptions to its journals for needy colleagues in Belarus, Poland, Slovakia and Ukraine. It also helps scientists who have immigrated to the United States by defraying some of the costs of their job searches and to facilitate their making professional contacts.

Quotation of the Day: “There are surely worse things than being wrong, and being dull and pedantic are certainly among them.” – Mark Kac