

## Heinz Hopf

One of Europe's leading topologists, **Heinz Hopf** (November 19, 1894 – June 3, 1971) worked on many aspects of combinatorial topology, including homotopy theory and vector fields. He was born at Gräbschen, near Breslau, then part of Germany, but now known as Wrocław, Poland. His early education took place at Dr. Karl Mittelhaus' higher boys' school from 1901 to 1904, and while attending the König-Wilhelm Gymnasium in Breslau his



mathematical ability became apparent. He entered the University of Breslau in 1913 and studied mathematics with Adolf Kneser, Erhard Schmidt, Max Dehn, and Rudolf Sturm. But his studies were interrupted by a long period of military service, serving on the Western front during WWI. He was wounded twice and received the iron cross, first class. While on a leave in 1917, Hopf visited Schmidt's set theory course, which convinced him to become a research mathematician.

After the war, Hopf returned to his studies at Breslau, moving on after a year to the University of Heidelberg, and then in 1920, he followed Schmidt to Berlin. Hopf received his Ph.D. in 1925 with a thesis on the topology of manifolds. He went to Göttingen in 1925 where he completed his habilitation thesis. Hopf made the acquaintance of the Russian mathematician Pavel S. Alexandroff, with whom he established a lifelong friendship. The two friends spent the 1927-28 academic year at Princeton University, where they collaborated with Solomon Lefschetz, Oswald Veblen and J.W. Alexander. While at Princeton, Hopf and Alexandroff began work on a planned three-volume work on Topology. The project did not go as hoped. The first volume didn't appear until 1935, and WWII prevented the completion of the remaining two volumes.

In 1928 Hopf married Anja von Mickwitz and in 1931 he filled the chair at the Eidgenössische Technische Hochschule in Zurich vacated by Hermann Weyl. Although Hopf's father had converted to the Protestant religion when he married, his family was Jewish. When the Nazis came to power, Hopf tried to arrange for his father to join him in Switzerland, but the elder man was too ill to travel. Hopf offered refuge to German friends fleeing the Nazis, and in 1943 the German government ordered Hopf to return to the Fatherland or lose his citizenship. Instead he chose to apply for Swiss citizenship, which was soon granted. The Nazis confiscated his property in Germany, but after the war Hopf helped reestablish the German mathematical community. He was offered several positions with prestigious universities, but chose to remain in Zurich.

Most of Hopf's research was in the area of algebraic topology, motivated by vigorous geometric intuitions. He extended Lefschetz's fixed-point theorem in a work begun in a 1928 paper, which was the first to use homology groups explicitly. While at Princeton, Hopf worked on the homology of manifolds and in 1931 defined what is now known as the "Hopf invariant" of maps between spheres of different dimensions that cannot be distinguished homologically. Hopf's work was instrumental in Witold Hurewicz's shaping the concept of homotopy groups of fiber spaces. Hopf also worked with families of vector fields and global differential geometry. His name is associated with many mathematical concepts, such as Hopf algebras, Hopf fibration of spheres, Hopf-Rinow complete Riemannian manifolds, and the Hopf theorem on the ends of groups. The Hopf fibration served as an important example, leading to the development of modern differential geometry and gauge theory, and is a fundamental object in the theory of Lie groups. It also has physical applications including magnetic monopolies, rigid body mechanics, and quantum information theory.

**Quotation of the day:** "The geometry of Algebraic Topology is so pretty, it would seem a pity to

slight it and miss all the intuition which it provides. At deeper levels, algebra becomes increasingly important, so for the sake of balance it seems only fair to emphasize geometry at the beginning.” –

Allen Hatcher