Jacques Hadamard

One of the last universal mathematicians, **Jacques Hadamard** (December 8, 1865 – October 17, 1963) was called "the living legend of mathematics." During his long life, he worked in many areas of mathematics but is probably best remembered for his 1896 proof of the prime number theorem, which was independently proved that same year by Belgian analyst and number theorist Charles de la Vallée Poussin. The theorem was a conjecture made by Gauss in 1792. It states that if $\pi(n)$ is the number of primes not greater than n, then $\pi(n)$ is asymptotic to $n/\ln n$, that is, the limit of their ratio is 1, as $n \to \infty$. Hadamard also introduced what have come to be known as Hadamard matrices that have found important applications in creating error correcting codes, experimental and combinatorial designs, masks for spectroscopic analysis, and in encryption. Private key encryption has become important in the modern era as the standard encryption for e-commerce, e-banking, and e-mail.

Hadamard was born in Versailles, France, where his father was a Latin teacher at the Lycée Impérial and his mother was a distinguished piano teacher. The family later moved to Paris where out subject lived as an infant through the prolonged and grueling siege of Paris during the Franco-Prussian War. The City of Lights was bombarded with new heavy caliber siege guns especially made for the battle by the Krupp Corporation. The population began to suffer from famine and was forced to kill their horses, cats, dogs and animals in zoos for food. On January 28, 1871, the French and the 146,000 defenders of Paris surrendered. Between the time of the humiliating French surrender and the signing of The Treaty of Frankfurt in May 1871, civil war broke out in Paris and the house occupied by the Hadamard family

was destroyed in a fire. During this period the family suffered further personal tragedies with the death of two of Jacques' sisters. In 1884 Hadamard entered the École Normale Supérieure, graduating in 1888. During the next five years he taught mathematics at various schools. He received his *docteur ès sciences* in 1892 for a thesis on functions defined by Taylor series. That same year, Hadamard married Louise-Anna Trénel who, like her husband, came from a Jewish family. They had five children.

The year 1892 was capped when Hadamard received the Grand Prix des Sciences Mathématiques for his paper "Determination of the number of primes less than a given number." Hadamard lectured at the Faculté des Sciences of Bordeaux from 1893 to 1897, during which time he published 29 papers on a wide range of topics, including his proof of the Prime Number Theorem. He moved to Paris in 1897 where he was a lecturer at the Sorbonne until 1909, the year he became professor at the Collège de France. For the next twenty years he ran the hugely successful *Séminaire Hadamard*. In 1912 Hadamard succeeded Camille Jordan as professor of analysis at the École Polytechnique. Tragedy struck during WWI when Hadamard's two oldest sons were killed at the Battle of Verdun. Hadamard later wrote that his many years of "pure joy" came to an end in that year. Hadamard retired from his various positions in 1937 at age 72.

When France fell to the German invaders in 1940, he and his family escaped to the United States, where he was appointed a visiting professor at Columbia University but could not find a permanent post in the United States. In 1944, he received the sad news that his third son had been killed in the war, while serving with the Free French forces in North Africa. Returning to France at the end of the hostilities Hadamard pursued his interests in music, ferns and fungi. He was such an active peace campaigner that it required the active lobbying of U.S. mathematicians to overcome political opposition to his attendance at the International Mathematical Congress held in Cambridge, Massachusetts in 1950. He was made honorary president of the Congress. A year before his death,

Hadamard experienced another painful tragedy. His beloved grandson Étienne was killed in a climbing accident. This appeared to be more than Hadamard could bear and he died a few months later, a bit short of his 98th birthday.

Hadamard published more than 300 novel and highly creative papers in analytic function theory, number theory, analytical mechanics, hydrodynamics, calculus of variations, differential geometry, partial differential equations, probability theory, logic, and the history of mathematics. At the end of the 19th century he proved one of the earliest results in what has come to be known as Chaos Theory, by showing why one can't predict what three billiard balls will do when they careen off each other on the table. In papers of 1901 and 1902, Hadamard classified all mathematical problems into either "well-posed" or "ill-posed." When a new problem is proposed, the first order of business is to establish that it has a solution, that the solution is unique, and that the solution depends in a reasonable way on the data. Such a problem is called "well-posed," because it is worth pursuing, as it is considered solvable. If a problem is not shown to have an existing and unique solution, it is considered "ill-posed," or unsolvable.

One of Hadamard's most fascinating investigations was in the psychology of mathematical invention, leading to the publication of his fascinating book *The Psychology of Invention in the Mathematical Field*. First published in 1945, it was reissued as *The Mathematician's Mind* in 1973. In it Hadamard explores how mathematicians invent new ideas by relating the creative experiences of great thinkers of the era. Hadamard held the opinion that mathematical thought is visual and that words only interfere. He included a letter from his close friend Albert Einstein analyzing his own mechanism of thought. Hadamard detailed how inspiration might occur at any time, often after an individual had struggled for days with a problem. Putting aside the problem and involving the conscious mind with other thoughts, the subconscious continues to work on the problem and much to the surprise and delight of the

individual the solution becomes clear. One of the intriguing conclusions of Hadamard's inquiry is that most of those he consulted did not "see" their problems in verbal terms or in algebraic symbols, but rather in visual images of a vague, hazy nature. Einstein wrote: "The words of the language as they are written or spoken do not seem to play any role in the mechanism of thought … which relies on more or less clear images of a visual and some of a muscular type."

Hadamard listed the stages of mathematical creativity as preparation (trial and error), incubation (often subconscious), illumination (frequently sudden), and verification (requiring reasoning). When trying to prove a theorem, the preparation stage involves an intense study of the problem, considering the use of various approaches common in the field. Once the problem is thoroughly understood, and known approaches exhausted, the mathematician may experience a sense of anxiety or frustration at being unable to attack the problem deductively. Incubation begins when the mathematician lives with the problem and allows unconscious processes to operate. If fortunate, illumination occurs, when a momentary flash of insight provides the solution. The last phase of invention is verification, which is when the mathematician writes out the deductive argument, whose structure appeared in the moment of illumination. Hadamard's contributions were important not only in determining how mathematical reasoning occurs, but if and how it can be taught. He emphasized that in most cases mathematicians can't foresee whether a tentative line of attack will be successful; but they have a sense of beauty that directs them. Once a young André Weil skipped a point in an argument, maintaining that it was obvious. Hadamard replied that either it was so obvious that it could be explained in a line, in which case the line should be included, or it was not obvious and needed to be explained.

Hadamard's first became involved in politics during the infamous "Dreyfus Affair." In 1894 Alfred Dreyfus (1859 – 1935), a French artillery captain of the General Staff, who was Jewish, was falsely accused of selling military secrets to the Germans. He was court-martialled, found guilty, stripped of

his rank, and sentenced to life imprisonment on Devil's Island in French Guiana. Dreyfus protested his innocence and had no apparent motive for treason. Although his trial had been highly irregular, anti-Semitism in France made the verdict popular. Hadamard was a second cousin of Dreyfus' wife Lucie, *née* Lucie Hadamard. Before the Affair, Hadamard and Dreyfus like most Jewish-French bourgeoisie considered them completely assimilated into French society. In March 1896 Colonel George Picquart of French military intelligence discovered that the document that had been the principal evidence against Dreyfus was not written in the hand of the accused, but of another officer, Major Esterhazy, a man with extensive gambling debts. Despite an attempt to hush of the discovery, the Dreyfus family became aware of this evidence and demanded that Esterhazy be tried for the crime. He was, but was acquitted by a military tribunal. Hadamard became a leader in the crusade to overturn Dreyfus's conviction and to clear his name. In 1898 French novelist Emile Zola espoused the cause of Dreyfus; publishing an open letter *J'accuse* in which he accused the army's General Staff of knowing that Dreyfus was innocent. He accused them of being willing to see a Jewish victim suffer rather than admit that they had used forged documents to unjustly convict Dreyfus. France was split into rioting camps. Zola was fined and sentenced to a year in prison, but escaped to England.

Eventually Dreyfus was retried, found guilty again, and was pardoned. Hadamard would not accept this result and worked diligently to clear Dreyfus' name, which occurred on July 2, 1906 when he was reinstated to his commission and decorated with the Legion of Honor. Two days after Dreyfus' death his funeral cortege passed the Place de Concorde through the ranks of troops assembled for the National Holiday. However it wasn't until 1995 that the French army officially admitted they had been wrong in the case. From the time of Dreyfus' vindication, Hadamard, who before the Affair had seen apolitical, became active in Jewish causes and the League for Human Rights, founded by Zola, and remained a public man for the rest of his life.

One of my professors enjoyed sharing stories of distinguished mathematicians he had known. In most cases, his stories appear in one source or another. Unfortunately, no confirmation of the following tale about Hadamard has been found. Perhaps, it's a mathematical urban legend. With this caveat, it's worth retelling. After completing a year as a visiting professor at Columbia, Hadamard sought another position in the United States while the war raged across the oceans. He had no luck. When he made a hasty exit from France in 1940, he didn't take his credentials with him. Each time he applied for a position, university officials were courteous, but insisted that without the necessary documentation they could not hire him. At one university, he recited his educational background, academic positions, publications, etc. to a dean. But the administrator said, "If you only had some sort of evidence of who you are and what you have done, I'd be happy to offer you a position." Hadamard asked for time to think on the matter and took a walk around the campus. He entered the library, looked around, and his face brightened. He hurried back to the dean's office. He asked, "Do you know those pictures of famous mathematicians on the walls of the library?" The dean answered, "Why, yes I do." Hadamard proudly announced, "I am the subject of the fourth picture to the left of the entrance." He still didn't get the position.

Quotation of the Day: "Some intervention of intuition issuing from the unconscious is necessary at least to initiate the logical work." – Jacques Hadamard