

## Kurt Gödel

In 1931, Austrian born U.S. mathematician **Kurt Gödel** (April 28, 1906 – January 14, 1978) made one of the major mathematical discoveries of the 20<sup>th</sup>-century. With his “Incompleteness Theorem” he showed that it is impossible to define a complete system of axioms, which is also consistent, that is, has no contradictions. As a result, any formal system that can generate meaningful statements is powerful enough to generate true statements that cannot be proved within the system. Consequently, mathematics cannot be placed on an entirely rigorous basis.



Gödel was born in Brünn (Brno), Moravia, then a part of Austria, now in the Czech Republic. He died in Princeton, New Jersey, when having developed paranoia that he was being poisoned, refused to eat and starved himself to death. Growing up, Gödel had been frequently ill and his health was a life-long concern. He entered the University of Vienna in 1923, and completed his doctoral dissertation under the supervision of Hans Hahn in 1929. The next year he joined the faculty at the University of Vienna, remaining there until 1938.

Gödel first presented his novel ideas at a Königsberg conference in 1930. This led in 1931 to his most famous paper, *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme* (*On Formally Undecidable Propositions of Principia Mathematica and Related Systems*), which contained the “Incompleteness Theorem,” and showed arithmetic to be incomplete. To a certain extent his theorem is a denial of the law of excluded middle, which is that a statement is either true or false. Samuel Delaney quoted Shakespeare, “There are more things in heaven and earth than are dreamed of in your philosophy” as an interpretation of the theorem. In *Mind Game* (1987), Rudy Rucker observed,

“...Gödel’s Theorem shows that human thought is more complex and less mechanical than anyone had ever believed...”

The Incompleteness Theorem has many profound implications, not just for mathematics but for philosophy as well. Gödel demonstrated that no matter how complete the analysis may seem to be, there would always be ideas outside the circle of any axiom system. Rucker likened the position of scientists as a result of Gödel’s theorem to Joseph K in Franz Kafka’s novel *The Trial*. They will be forced to scurry around seeking answers in a labyrinth with no way of escaping. In an article that appeared in *The Guardian* (April 26, 2001), Keith Devlin reported that some scientists such as Roger Penrose use the Incompleteness Theorem to argue that the human brain does not operate like a computer, and that artificial intelligence is impossible. According to Penrose’s interpretation of Gödel’s Incompleteness Theorem, mathematics has an element that is completely creative.

Gödel’s theorem ended a hundred years of attempts to derive logically the whole of mathematics from a logical axiom system. It demonstrated the impossibility of ever having a computer programmed to answer all mathematical questions. What is often overlooked is that in his doctoral dissertation Gödel showed that all true statements in the lower predicate calculus could be proved within the system. Lower predicate calculus is the system of logic that includes propositional functions, relationships and quantifiers. Although it does not cover all the reasoning used in mathematics, it is the system most often used by modern logicians. The result is sometimes referred to as Gödel’s “Completeness Theorem.”

When Hitler came to power in 1933, Gödel was initially unaffected. But when a Nazi student murdered Gödel’s former teacher Mortiz Schlick, Gödel suffered his first nervous breakdown. Soon after his recovery, he was invited to join the Institute for Advanced Study at Princeton. There he gave a series of

lectures on the subject “On undecidable propositions of formal mathematical systems.” At Oswald Veblen’s suggestion, Stephen Kleene took notes of the lectures, which were initially circulated in mimeographed form, but were subsequently published under his name. In 2001, Oxford University Press published *Kurt Gödel’s Collected Works: Unpublished Essays & Lectures*, which includes the notes.

He returned to Austria and married nightclub dancer Adele Porkert Nimbursky in 1938.

After the *Aeschylus* in 1938, Austria became a part of Nazi Germany. In 1940, fearing conscription into the German army, Gödel and his wife left for the United States via the Trans-Siberian railway. They settled in Princeton, where he resumed his membership at the Institute for Advanced Study. He continued his work on logic and that same year published his classic paper *Consistency of the axiom of choice and of the generalized continuum-hypothesis with the axioms of set theory*. In the late 1940s he demonstrated the existence of paradoxical solutions to Einstein’s field equations in general relativity. In 1948 Gödel became a permanent member of the Institute and was made a professor at the Institute five years later, a position he held until his retirement in 1976.

In 1949, when Gödel applied to become a U.S. citizen, he had an impressive list of sponsors: Albert Einstein, Oskar Morgenstern and John von Neumann. They were a bit apprehensive about how Gödel would handle the simple citizenship interview with a judge. All he had to do was answer a few elementary questions about the American Constitution. On the eve of the interview, Gödel announced that he had discovered a logical loophole that would enable a dictatorship to be established. He was told that this was absurd, but his friends were concerned that he would bring this up in the interview and be denied citizenship. John Casti, quoted in John D. Barrow’s *Pi in the Sky* (1992), gives the following account of what happened then:

“At the interview the judge was suitably impressed by the sterling character and public personas of Gödel’s witnesses, and broke with tradition by inviting them to

sit in during the exam. The judge began by saying to Gödel, “Up to now you have held German citizenship.” Gödel corrected this slight affront, noting that he was Austrian. Unfazed, the judge continued, “Anyhow, it was under an evil dictatorship ... but fortunately that’s not possible in America.” With the magic word dictatorship out of the bag, Gödel was not to be denied, crying out, “On the contrary, I know how that can happen. And I can prove it!” By all accounts it took the efforts of not only Einstein and Morgenstern but also the judge to calm Gödel down and prevent him from going into a detailed and lengthy discourse about his “discovery.”

After his death in 1978, Gödel’s private papers revealed his eccentric and idiosyncratic nature. They contained boxes of unanswered letters together with drafts of replies never sent. During his years at Princeton, Gödel never enjoyed being the focus of attention or encountering controversy. He went to unusual extremes to avoid meeting people. If someone wished to see him, Gödel did not decline but made an appointment and then failed to show up. He was a very religious person and even wrote an elaboration on Leibniz’s ontological proof of God’s existence, which he shared in a letter to his mother.

**Quotation of the Day:** “Either mathematics is too big for the human mind or the human is more than a machine.” – Kurt Gödel