Pierre de Fermat

French lawyer, mathematician and physicist **Pierre de Fermat** (August 17, 1601 – January 12, 1665) is known as the "Prince of Amateurs." His name will always be associated with his likely mistaken claim of having found a proof of his famous "Last Theorem," which asserted the impossibility of expressing any number that is a power greater than two as the sum of two like powers. He is more deserving of being remembered for developing the modern theory of numbers; discovering the basic principles of analytic geometry, independently of René Descartes; finding equations for tangent lines to curves and a method of determining minima and maxima points of curves; and establishing a method for the quadrature of parabolas simpler than that due to Archimedes, thus laying the foundations upon which Isaac Newton built his infinitesimal calculus; sharing with Blaise Pascal the founding of the theory of probability; and advancing the principle that light when reflected, always takes the path for which the travel time is the least. From this "principle of least time," the law of refraction and the law of reflection were deduced. Fermat did not publish his work in the usual sense. Only one major manuscript appeared during his lifetime. His results were contained in voluminous correspondences with friends, notes in the margins of his books, and challenges to others to find proofs of theorems he devised. He enjoyed the pleasure of discovery more than any reputation his work might bring him.

Fermat was born in Beaumont-de-Lomagne, the son of a wealthy leather merchant. Of his early life and education little is known with certainty. It seems that he received a classical secondary education first at the convent of the Cordeliers in Beaumont, run by the Franciscans, and then studied with the Jesuits. Fermat attended the University of Toulouse where he began his first serious mathematical research,

which led to his methods for finding tangents to curves and their maximum and minimum points. He moved to Bordeaux some time in the 1620s and studied jurisprudence at the University of Orleans, receiving a Bachelor of Civil Laws in 1631. That same year he purchased the offices of councilor at the parliament in Toulouse, and married his fourth cousin Louise de Long, with whom he had five children. Fermat received many governmental appointments, and although he appears to have been an indifferent lawyer, he was a successful magistrate. He received many promotions, in part because many of the older men in the courts were killed by plagues.

The mathematical theory of probability began as a result of Fermat's correspondence with Blaise Pascal. Gambler Chevalier de Mere questioned the common assumption that a double six would appear in 24 rolls of a pair of dice. He also asked Pascal how money at stake should be divided among equally skilled players should they stop playing before the game is completed. Pascal sought Fermat's opinion and in a series of five letters, they agreed that the probability is equal for all outcomes, but one must determine all the probable outcomes, and then calculate how many are favorable to the desired outcome. Fermat's method of arriving at the answer was simpler than Pascal's, being built upon determining combinations.

Fermat had a long rivalry with Descartes that originated with some critical remarks he made about Descartes' work "La Dioptrique." Fermat claimed Descartes had not correctly deduced the law of refraction. Descartes was furious and did as much as he could to damage Fermat's reputation, asserting that Fermat was inadequate as a mathematician and as a thinker. He attacked Fermat's method of maxima, minima and tangents, which led to a heated argument, eventually referred to Girard Desargues to referee. Fermat was proved to be correct and Descartes finally admitted this in writing. Despite Descartes' efforts, Fermat's reputation as one of the leading mathematicians of his time was firmly established, even though he resisted attempts to convince him to prepare his work for publication. His

work was not as highly regarded as it deserved to be because he gave no expositions of his methods, and he used the awkward notation of François Viète, which had been made obsolete by Descartes' notations used in his *Géométrie*.

Fermat's major mathematical interest was in number theory. Some of the most valuable of his results were discovered after his death on loose sheets of paper or in the margins of books he had read and annotated. His friends feared that since he had not published anything his work would be forgotten after his death. His son Clement-Samuel collected his father's letters, mathematical papers, and comments written in books, a total of some 3000 mathematical items. The *Opera mathematica* of Fermat were published in two volumes in 1670 and 1679. It was in the margins of his copy of the now lost Claude Bachet's translation of Diophantus' *Arithmetica* that his famous "Last Theorem" appears. He claimed that there are no positive integers x, y and z such that, $x^n + y^n = z^n$, for integers $n > 2$. Fermat went on to say: "I've found a remarkable proof of this fact, but there is not enough space in the margin to write it."

In 1994 Andrew Wiles successfully proved the theorem that had fascinated many professional and amateur mathematicians in the intervening years. As to Fermat's claim of having found "a remarkable proof," it is unlikely. He may have discovered a method for proving the cases when n=3 and n=4 and became convinced that similar proofs would work in the general case. Another of Fermat's publicized conjectures turned out to be false. He asserted his belief that numbers of the form 1 more than 2 raised to the power 2^n were prime numbers. He seems to have been convinced when this was shown to be the case for $n < 5$. A century later Euler demonstrated the so-called *Fermat number* F(n) was not prime when $n = 5$, in fact $2^{32} + 1 = 4{,}294{,}967{,}297 = 641 \times 6{,}700{,}417$.

Later Carl Friedrich Gauss found a use for divisors of Fermat numbers when he showed that a regular

polygon with n sides is constructible with straightedge and compass if n is a prime Fermat number or a product of different Fermat primes. Fermat's claim of having found a proof of his "Last Theorem" was only a private comment, which wasn't corresponded to other mathematicians nor did he offer it as a challenge to others as he often did with results he had already proved. Perhaps Fermat discovered a flaw in the proof he believed he had, and never crossed out his comment because he never expected anyone to see it.

Fermat was the first to use a method of proof called the "infinite descent." It is a particular type of proof by mathematical induction. A typical application is show that no positive integer exists with a certain property. Assume the contrary, that some positive integer x has the property. Next, deduce that there is some positive integer $y < x$ that also has the property. Repeat this argument indefinitely thus infinitely descending through all positive integers. Then one must show that this infinite descent implied by having a whole sequence of solutions that are even smaller, by our chosen measure, is impossible. This is a contradiction so no positive integer exists with the given property. In a letter quoted in Weil's *Number Theory*, Fermat described his use methods:

> "As ordinary methods, such as are found in books, are inadequate to proving such difficult propositions, I discovered at last a most singular method … which I called the infinite descent. At first I used it only to prove negative assertions… To apply it to affirmative questions is much harder, so when I had to prove 'Every prime of the form $4n + 1$ is the sum of two squares," I found myself in a sorry plight (en belle peine). But at last such questions proved amenable to my methods."

During the period from 1643 to 1654, Fermat's correspondence with other mathematicians ceased in part because of a civil war in the country greatly affecting Toulouse. Further the plague struck the

region in the 1650s. Fermat died in Castres, near Toulouse, in his sixty-fifth year. It wasn't until a hundred years later that Euler succeeded in giving proofs of most of the number theory problems that Fermat claimed to have solved but for which had not recorded his proofs.

Quotation of the Day: "I have so little aptitude in writing out my [mathematical] demonstrations that I have been content to have discovered the truth, and to know the means of proving it when I shall have reason to do so." – Pierre de Fermat