

## [JOHANN] PETER GUSTAV LEJEUNE DIRICHLET

[Johann] Peter Gustav Lejeune Dirichlet (February 13, 1805 – May 5, 1859) developed significant theorems in the areas of elliptic functions and applied analytic techniques to mathematical theory that resulted in the fundamental development of number theory. His most important work was on the convergence of the Fourier series. He also applied his mathematical results to physics, such as the analysis of vibrating strings and wrote a critique of the ideas of the stability of the solar system as proposed by Pierre de Laplace.



Dirichlet was born in Duren, then a part of the French Empire, but now in Germany. His family name originates from “Le jeune de Richelet” which means “Young from Richelet.” Richelet was a town in Belgium where his grandfather lived. Dirichlet had a passion for mathematics from an early age and spent whatever money he could raise on mathematics books. He attended the gymnasium in Bonn for two years and then the Jesuit College in Cologne, where George Simon Ohm taught him mathematics and science. At age 16 Dirichlet completed the requirements for entrance to the University, but at the time, with the exception of Carl Friedrich Gauss at Göttingen, most of the mathematicians at German universities were not eminent. Dirichlet journeyed to the Collège de France in Paris, attracted by the chance to learn mathematics from the likes of Jean Baptiste Fourier, Pierre Simon Laplace, Sylvestre François Lacroix, Adrien Marie Legendre, and Siméon Denis Poisson. Dirichlet took with him his most prized possession, a copy of Gauss’ *Disquisitiones arithmeticae*, which he treated like a holy book, keeping it with him always, reportedly sleeping with his copy under his pillow at night. The admiration was mutual. Gauss once remarked, “The total number of Dirichlet’s publications is not large, but jewels are not weighed on a grocery scale.”

In 1823, Dirichlet became German tutor to the wife and children of General Maximilien Sebastien Foy, a major figure in the army during the Napoleonic Wars. He lived in the Foy home and was treated as a member of the family. Dirichlet presented his first memoir to the French Academy of Sciences in 1825. His proof of a special case of Pierre Fermat's Last Theorem, namely that there are no integers  $x$ ,  $y$  and  $z$  for which the sum of the fifth powers of  $x$  and  $y$  is equal to the fifth power of  $z$ , brought him instant fame. Dirichlet divided the proof for the fifth power into two cases. The first was proved in his memoir to the Academy, reportedly based on an idea suggested by Sophie Germain, and a short time later Legendre published a proof for the second case in an appendix to his book *Théorie des nombres*. Two months later Dirichlet supplied a more elegant proof of the second case, a natural continuation of the proof of the first case.

After Foy's death, Dirichlet returned to Germany but could not find a university position because he had no doctorate. This problem was solved when his friend Alexander von Humboldt arranged for the University of Cologne to award Dirichlet an honorary doctorate for his mathematical work. Dirichlet taught at the University of Breslau in 1827 and at the University of Berlin from 1828 to 1855. Upon Gauss' death in 1855, Dirichlet succeeded the mathematician whose work he most admired as professor of higher mathematics at Göttingen. His lectures on the equilibrium of systems and potential theory led to what is known as the Dirichlet problem. It involves finding solutions to differential equations for a given set of values of the boundary points of the region on which the equations are defined. The problem is also known as the first boundary-value problem of potential theorem. After his death one of his students published Dirichlet's lectures, the first book on the subject.

In addition Dirichlet proved the Gaussian number theory conjecture that in any arithmetic progression

with first term relatively prime to the difference between consecutive terms there are infinitely many primes. Analytic number theory began with Dirichlet's memoir on this problem. Among his other works, Dirichlet wrote valuable papers on conditions for the convergence of trigonometric series and the use of series to represent arbitrary functions. Dirichlet is also given credit for introducing the modern concept of a function, writing in 1837:

“If variable  $y$  is so related to a variable  $x$  that whenever a numerical value is assigned to  $x$ , there is a rule according to which a unique value of  $y$  is determined, then  $y$  is said to be a function of the independent variable  $x$ .”

Besides his own research, Dirichlet planned to finish Gauss's incomplete works but because of his early death he was only able to publish a few such memoirs. J.W.R. Dedekind edited Dirichlet's lectures on number theory and published them as *Vorlesungen über Zahlentheorie*. Dirichlet fleshed out Gauss's concise and rigid proofs in his *Disquisitiones arithmeticae*, simplifying them and transforming the ideas into essentially modern form, making the results more available to other mathematicians.

Dirichlet married Rebecca Mendelssohn, the granddaughter of the philosopher Moses Mendelssohn, and a sister of the composer Felix Mendelssohn. Dirichlet rarely wrote letters, even to close friends, but he announced the birth of his first child to his father-in-law with the simple message “ $2 + 1 = 3$ .” In 1858 during a conference in Montreux, Switzerland, Dirichlet suffered a heart attack and returned to Göttingen gravely ill. About this time his wife died, increasing his misery, and he died soon after. Although German university education in mathematics was not of a high quality when Dirichlet was a student, he shares credit with his many outstanding research students, including Kronecker, for making German universities the best mathematical centers in the world.

Quotation of the Day: “In mathematics as in other fields, to find oneself lost in wonder at some manifestation is frequently the half of a new discovery.” – J.P.G. Lejeune Dirichlet