Alexis Claude Clairaut

French analyst, differential geometer, and astronomer **Alexis Claude Clairaut** (May 7, 1713 – May 17, 1765) was one of the most renowned mathematicians and physicists of the 18th century. His father, Jean-Baptiste Clairaut, who taught mathematics in Paris, set very high expectations for his precocious son. Alexis's poor mother, Catherine Petit, gave birth to twenty children, with only Alexis surviving to adulthood. Alexis was taught to read by his father using Euclid's *Elements* as a primer. By age ten he had mastered the textbooks of L'Hôpital on the calculus and conics. At thirteen he read a paper on geometry, *Quatre problèmes sur de nouvelles courbes,* to the .

In 1731 Clairaut published *Recherche sur les courbes à double courbe*, in which he analytically treated fundamental problems of curves in space. He referred to these space curves as "tortuous" or "curves of double curvature," because they do not lie in a single plane and their curvature is determined by the curvature of two projections. Both René Descartes and Pierre de Fermat knew that first-degree curves, represented by equations of the form $ax + by + c = 0$, were straight lines and that second-degree curves, represented by equations of the form $ax^2 + bxy + cy^2 + dx + ey + f = 0$, were conic sections. Isaac Newton was the first to investigate systematically third degree or cubic equations. His work on the subject, *Enumeratio Linearii Tertii Ordinii*, completed in 1676, was not published until 1704. In it he claimed that all curves represented by the general third-degree equation,

$$ax^3 + bx^2y + cx^2 + dy^3 + exy^2 + fxy + gy^2 + hx + jy + k = 0$$

could be reduced by an appropriate choice of coordinate axes to one of four simpler forms.

Newton also claimed that all cubic equations could be obtained from those of the form,

$$y^2 = ax^3 + bx^2 + cx + d \quad (1)$$

by projecting between planes. In 1731, Clairaut and Françoise Nicole independently published proofs of Newton's assertion. Geometrically, Clairaut thought of a space curve as the intersection of two surfaces; and analytically the equation of each surface was expressed as an equation in three variables. He introduced a surface in three-space, defined by the equation

$$zy^2 = ax^3 + bx^2z + cxz^2 + dz^3 \quad (2).$$

Clairaut established Newton's claim by proving that every cubic curve is the intersection of a plane and a cubic cone of the form (2). In Clairaut's treatise, the first work on solid analytic geometry, he gave distance formulas for two and three dimensions, an intercept form of the plane, and found tangent lines to space curves.

Clairaut's work led to his election to the Académie des Sciences, with the rules of admission being suspended to accommodate the remarkable prodigy at the unprecedented age of seventeen. There he joined a small group of young mathematicians, led by Pierre Louis Maupertius, who supported the natural philosophy of Newton. Not all mathematicians of the time accepted Newton's findings in the *Principia Mathematica*. Euler never accepted Newtonian gravity, even as he proceeded to perfect Newtonian lunar theory, based on discoveries made by Clairaut. Our subject was a close friend of Voltaire and his mistress Emile de Breteuil, Marquise du Châtelet. He became the Marquise's tutor and helped her translate the *Principia* into French.

In 1741 Clairaut accompanied Maupertius on a scientific expedition to Lapland in the Artic circle to collect data to be used in measuring the length of a meridian degree on the earth's surface. A similar expedition led by Charles Marie de la Condamine measured the equatorial curvature in the Andes. The purpose of the expedition was to determine the shape of the earth by measuring its curvature at the places where it differed most – the equator and the poles and thus to test Newton's assertion that the

Earth's rotation should cause it to bulge at the equator and flatten at the poles. Cartesian theory predicted just the reverse. On his return Clairaut produced his classic work *Théorie de la figure de la Terre* ("Theory and Shape of the Earth,"1743) in which he calculated the form a rotating body assumes because of the mutual attraction of its parts. Clairaut reported that as Newton had predicted, the Earth has a larger diameter through the equator than through the poles, a shape known to geometers as an oblate spheroid. In 1749, using Newton's laws, Clairaut showed that the north pole of the Earth precesses (change in direction of the earth's axis) with a period of 25,800 years. So, some 13,000 years from now, the North Pole will not be pointing towards Polaris, the "North Star," but will be pointing close to the star Vega. This will happen because the Sun and Moon exert a torque on the equatorial bulge, of the Earth. This causes the axis of rotation of the Earth to wobble. To get an idea of a picture of this, think of a spinning top as it tilts to the side as its spinning slows down.

Clairaut's work *Théorie de la lune* (1752) contained the explanation of the motion of the apse - a semicircular or polygonal projection of a plane - that previously had puzzled astronomers. In his treatise he gave an argument explaining why he believed Newton's theory of gravity was incorrect. He redid Newton's calculation, found an error in the latter's derivation, and corrected it. Clairaut subsequently wrote various papers on the orbit of the moon, and published *Théorie du mouvement des comètes* ("Theory on the Movement of Comets" 1760) in which he predicted the date that Halley's comet would be at perihelion, the point it would be nearest the sun, was April 13, 1759. The actual date was March 13, just within the allowed-for margins of error. There was a suggestion that Halley's Comet be renamed for Clairaut and he was hailed as the "new Thales." Clairaut's *Eléments d'algèbre* (1749) was used as a text in French schools for many years. He studied a family of ordinary differential equations that are named for him and showed the equality of mixed partial derivatives. After a brief illness, Clairaut died at age 52 at the height of his powers.

Quotation of the Day: "It is not surprising that Euclid goes to the trouble of demonstrating that two circles which cut one another do not have the same center, that the sum of the sides of a triangle which is enclosed within another is smaller than the sum of the sides of the enclosing triangle. This geometer had to convince obstinate sophists who glory in rejecting the most evident truths; so that geometry must, like logic, rely on formal reasoning in order to rebut the quibblers." – Alexis Claude Clairaut