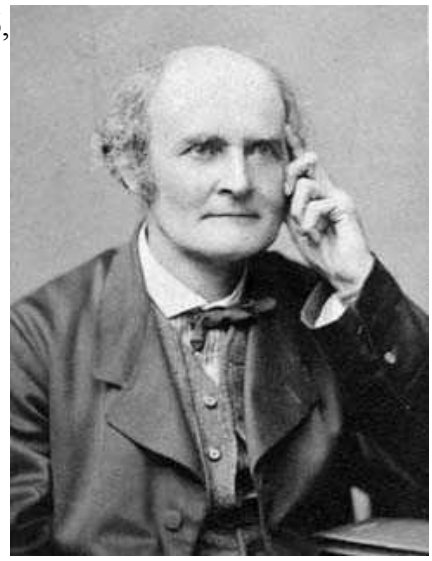
Arthur Cayley



English mathematician **Arthur Cayley** (August 16, 1821 – January 26, 1895), one of the greatest and most prolific mathematicians, helped found the British school of pure mathematics. He was one of the first to realize how many different areas of mathematics could be drawn together by the theory of groups. Together with James Joseph Sylvester, Cayley established the theory of algebraic invariants, a concept important in modern physics, particularly in relativity theory. Cayley blazed the trail for Felix Klein's discovery that Euclidean geometry along with the non-Euclidean geometries of Lobachevsky and Riemann were only aspects of a more general kind of geometry, including them as special cases.

Although born at Richmond in Surrey, Cayley spent the first eight years of his life in St. Petersberg, Russia where his successful merchant father and his mother had settled. When the family returned to England they settled at Blackheath, near London, where Cayley enjoyed solving complex mathematics problems for his amusement. He attended King's College School, where his mathematical acumen was so great he was encouraged to pursue studies in mathematics rather than enter the family business, as his father desired. In 1838 Cayley entered Trinity College, Cambridge, where he excelled in Greek, French, German, and Italian, as well as mathematics. The next year, he graduated as Senior Wrangler and won the first Smith's prize. He won a Fellowship and while tutoring at Cambridge for three years, published 28 papers in the newly founded *Cambridge Mathematical Journal*. Needing a more secure and remunerative profession than provided by the limited tenure of a Fellow, he entered Lincoln's Inn to prepare to be a solicitor. Three years later he was admitted to the bar.

During his 14-year law career Cayley specialized in conveyancing, the act of drawing up documents for transferring the ownership of real property. Though he found this boring, the position left him plenty of time to pursue his mathematical interests. While practicing law he published some 250 mathematical papers; small potatoes in comparison to what was to come. By the time of his death Cayley published one book and nearly a thousand mathematical papers, many of which are considered classics in various fields of pure mathematics, theoretical dynamics and astronomy. His collected works were published at Cambridge in 13 large quarto volumes of about 600 pages each, seven of which he edited himself. The remaining were edited under the supervision of Cayley's former student, biographer, and successor as Sadleirian professor at Cambridge, A.R. Forsyth. Cayley is the fourth most prolific mathematical writer in history, surpassed only by Euler, Erdős and Cauchy.

Before being called to the bar, Cayley visited Dublin to hear Sir William Rowan Hamilton lecture on quaternions and in 1854 Cayley showed that quaternions could be applied to the representations of rotations in four-dimensional space. The idea of a group gradually was formed from various mathematical problems, most importantly the study of the solution of algebraic equations by Niels Abel and Évariste Galois. Cayley gave the modern definition of a group, a set with a binary operation, emphasizing that a group is determined by the rules (axioms) governing its elements. Up to this time, those who spoke of groups, meant permutation groups. *Cayley's Theorem* states that every finite group is isomorphic to a subgroup of a permutation group.

In 1863, when the requirement of taking holy orders was dropped, Cayley accepted the newly established Sadleirian Chair of Pure Mathematics at Cambridge. Despite a marked drop in salary, he spent the rest of his life in the position. One of his proudest achievements during this time was in seeing his goal of having women admitted to Cambridge fulfilled, which seems only proper, as his chair had been established by provisions in the will of Lady Sadleir. At forty-two Cayley married Susan

Moline. His marriage and home life were quite happy. The couple had a son and a daughter. In 1883 Cayley became the President of the British Association for the Advancement of Science. He delivered one of the most memorable presidential addresses in which he gave an excellent account of his personal views on mathematics. He remained creatively active up to the end, which came on January 26, 1895 after a long and painful illness.

Cayley developed a theory of metrical geometry, dependent on sizes of angles and lengths of lines. He wrote, "… the metrical properties of a figure are not the properties of the figure *per se* … but its properties when considered in connection with another figure, namely the conic termed the absolute." Cayley and Klein showed that non-Euclidean geometries could be derived as special cases of projective geometry. They classified geometries as elliptic or hyperbolic depending on the curvature of space upon which the geometry was drawn, that is, whether a surface in the space is saddle-like or dome-like. Cayley's development of n-dimensional geometry has been applied in physics to the study of the Space-Time Continuum and his theory of matrices served as a foundation for Werner Heisenberg's revolutionary elucidation of quantum mechanics.

Quotation of the Day: "As for everything else, so for a mathematical theory: beauty can be perceived but not explained." – Arthur Cayley