GEORG CANTOR

Russian-born German mathematician **Georg Cantor** (March 3, 1845 – January 6, 1918) sparked a profound revolution in mathematical and philosophical thought when he promulgated his theory of transfinite numbers. His theory of infinite sets became a building block of modern mathematics, revolutionizing almost every mathematical field. However, he paid a great price for his accomplishment, when powerful mathematicians, who vigorously opposed his original views, made Cantor likely the most personally and professionally assailed mathematician in all history.

Cantor was born in St. Petersburg, Russia, the son of a successful merchant and an artistic mother. His father, who had been in poor health, moved his family to Germany to find a warmer climate. Cantor's father wanted his son to become an outstanding engineer, but Georg was more interested in mathematics. After his father's death, Cantor entered the University of Berlin, where he received his doctorate in 1867 with a dissertation on number theory. Unable to secure a position with one of the prestigious universities, Cantor accepted a position at Halle, where he turned his attention to research in analysis. He published a paper on trigonometric series in 1872 in which he defined irrational numbers in terms of convergent sequences of rational numbers. While studying a problem in analysis, Cantor looked into its "foundations," especially sets and infinite sets. What he found flabbergasted him and caused him to tell a friend: "I see it but I don't believe it." Cantor moved into the work that would bring him his greatest fame and greatest frustration.

Between 1879 and 1884 Cantor published a series of papers designed to provide a basic introduction to set theory. He is credited with creating the subject and putting forth a modern theory of infinite sets that

changed the course of mathematics. There had been a long history of mathematicians and philosophers avoiding infinity as a mathematical value. Aristotle, in his *Physics*, insisted the infinite had only a potential existence, not an actual one. In 1831, Carl Friedrich Gauss held that infinite magnitude might never be used as something final, but only as "a way of speaking." The fifth paper of Cantor's series, *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (Foundations of a General Theory of Manifolds*)* was his reply to his critics, who held views in sharp contrast to his about numbers and the infinite.

Cantor considered infinite sets as more than something that went on forever. He believed such sets had an actual though infinite number of members. He called these actual infinite numbers transfinite numbers. In 1873, Cantor proved the rational numbers are countable. That is, they can be put into one-to-one correspondence with the natural numbers. He was also the first to prove the real numbers were not countable, that is, they could not be put into one-to-one correspondence with the natural numbers. He not only considered infinite sets as existing totalities, but went so far as to distinguish them by introducing definitions and means of determining if two infinite sets contained the "same number of elements" or if one contained "more" than another. He showed that the set of points of a line segment has the same transfinite number as the set of points of an infinite line, even that of a plane. Cantor demonstrated that the set of algebraic numbers was "smaller" than the set of transcendental numbers.

To many, Cantor's papers published in 1874 and 1878 were nonsense at best and mathematical heresy at worst. Allegedly, Henri Poincaré claimed that later generations would consider set theory "a disease from which one has recovered." But the most vicious personal attacks on Cantor and his work came from Leopold Kronecker, who held that the only numbers were whole numbers, dismissing negatives, fractions, imaginary numbers, and especially irrational numbers as figments of diseased minds, having no place in mathematics. Kronecker considered Cantor a charlatan and used his considerable prestige to suppress Cantor's ideas and prevented him from achieving his goal of being appointed a professor at

the University of Berlin. The attacks on Cantor's work seemed to be contributing factors to his bouts of depression and several mental breakdowns during the remaining 33 years of his life, placing him alternately in and out of mental institutions; he died in one in 1918. Today Cantor's work is considered one of the greatest achievements of human reasoning. Unfortunately the recognition he so greatly deserved came too late for him to enjoy it. Cantor never doubted that his work was correct, but when paradoxes were discovered in set theory, he left the defense of transfinite set theory to younger mathematicians such as David Hilbert, Bertrand Russell, and Ernst Zermelo.

When Cantor considered the set of all sets, he showed that the set of all subsets of a set must have a larger transfinite number than the set itself. This led to the paradox that there must be a transfinite number larger than the largest one. He concluded that it was necessary to make a distinction between what he called "consistent" and "inconsistent" sets. In an 1899 letter to Richard Dedekind, he declared that the set of all sets or its number could not be considered.

Quotation of the Day: "No one shall drive us from the paradise that Cantor created for us." – David Hilbert