George David and Garrett Birkhoff

Today two American mathematicians, **George David Birkhoff** (March 21, 1884 – November 12, 1944) and his son **Garrett Birkhoff** (January 19, 1911 – November 22, 1996) are featured. The elder Birkhoff was born in Overisel, Michigan, the son of a physician who had come from Holland in 1870. When Birkhoff was two, the family moved to Chicago. From 1896 to 1902, he studied at the Lewis Institute (now the Illinois Institute of Technology). Following a year at the University of Chicago, he transferred to Harvard University. While still an undergraduate G.D. Birkhoff collaborated with Harry Vandiver on a problem in number theory, "On the integral divisors of $a^n - b^n$," which was published in 1904 in the *Annals of Mathematics*. G.D. earned a bachelor's degree in 1905 and a master's degree in 1906 from Harvard, before returning to Chicago to study for his doctorate. He wrote a dissertation "Properties of Certain Ordinary Differential Equations with Applications to Boundary Value and Expansion Problems" under the supervision of Eliakim Hastings Moore and was awarded a Ph.D. *summa cum laude* in 1907.

Between 1907 and 1909, G.D. taught at the University of Wisconsin. During this period he married Margaret Elizabeth Grafius. They had three children, Barbara, Garrett, and Rodney. G.D. took a position as an assistant professor at Princeton University, where he was involved in the exploratory studies of *analysis situs*, soon to be renamed "topology." Birkhoff joined the faculty of Harvard

University in 1912. After Henri Poincaré's death in 1912 Birkhoff took up the leadership in the field of dynamics. He received Poincaré's last paper, in which Poincaré showed how the existence of periodic solutions of the restricted problem of three bodies could be deduced from a rather simple geometric figure. However, anticipating his death, Poincaré had time only to prove the theorem in certain special cases. By the end of the year, Birkhoff gave a simple proof of what is now known as the Poincaré-Birkhoff fixed-point theorem.

Birkhoff's mathematical research was mainly in the fields of analysis applied to dynamics. He made notable contributions to differential and difference equations and celestial mechanics. In 1931 he proved the ergodic theorem, which transformed the Maxwell-Boltzmann ergodic hypothesis of the kinetic theory of gases into a rigorous principle. Ergodic theory is the study of measure-preserving transformations - in particular, the study of theorems concerning the limits of probability and weighted measures. It deals with questions of long-term equilibrium and stability of a varying system. It had its origins in much of the classical theory of statistical mechanics, exploring such thermodynamic properties as temperature and entropy. His success made Birkhoff the most famous American mathematician of his time. He wrote extensively about the theory of relativity and quantum mechanics, and developed a theory of gravitation, although his ideas in this field were not widely accepted. In the early 1920's, he published a paper "A set of postulates from plane geometry, based on scale and protractor." The difference between his axiom system for geometry and earlier ones is that Birkhoff's axioms made use of the real numbers and their properties to introduce the concepts of distance, angle measure, and area. In the early 1960's, the School Mathematics Study Group (SMSG) modified Birkhoff's axiom system to serve as a new standard for teaching high school geometry.

Birkhoff's lifelong interest in music and the arts led to his development of a mathematical theory of aesthetics, which he detailed in *Aesthetic Measure* (1933). Birkhoff conceived the ambitious goal of

creating a "general mathematical theory of the fine arts, which would do for aesthetics what had been achieved in another philosophical subject, logic, by the symbolisms of Boole, Peano, and Russell." He held that although the aesthetic feeling is intuitive, nevertheless the attributes on which aesthetics rests are measurable. The three main variables, as identified by Birkhoff, were: the *complexity* (C) of the object, the feeling of value or *aesthetic measure* (M), and the property of harmony, symmetry or order (O). He developed his theory to a point that he identified the basic formula, M = O/C, which asserts that the aesthetic measure is determined "by the density of order relations in the aesthetic object." It is necessary to read Birkhoff's work to discover how he mathematically formulates the problem of the aesthetic experience, and precisely what he means by "complexity" and "order." An obvious problem with Birkhoff's ratio is that it is very difficult to define its two constituents in an operational and meaningful way. Nevertheless, encouraged by his "successes in taking a mathematical approach to measuring aesthetics, G.D. moved on to a mathematical approach to ethics, which he considered to be analogous, in many respects. His formula in this case is even simpler. M (ethical measure) = G (total good achieved). He argued that the "ethically-minded person endeavors always to select that one of the possible courses of action which *maximizes* the ethical measure G."

Birkhoff has been accused of systematically keeping Jews out of his department at Harvard. Einstein claimed: "G.D. Birkhoff is one of the world's great anti-Semites." Throughout his life, Birkhoff argued that Einstein's general relativity was an unhelpful theory. Reportedly, Birkhoff helped some Jewish mathematical refugees find positions at American universities in the 1930s, but not at Harvard. He was dedicated to the growth and development of what he called "American mathematics," and always was on the lookout for talented young mathematical aspirants to contribute to his "cause." Oswald Veblen described Birkhoff's efforts in this regard as "a sort of religious devotion." To further his goal, he accepted the public responsibilities that came his way. He was very active with the American Mathematical Society, serving as its president between 1924 and 1926. He died in his sleep on

November 12, 1944, a victim of a weak heart.

Born in Princeton, New Jersey, Garrett Birkhoff was educated at home until he was eight years old. He then spent three years at a public grammar school, but took a year off before entering high school to play sports and mature. He was only twelve when he entered the private Browne and Nichols high school, where he excelled in mathematics under the tutelage of Harry Gaylord. Gaylord, an excellent mathematics teacher, had co-authored a textbook with Maxime Bôcher, a former teacher of Garrett's father at Harvard. Garrett whizzed through his studies in three years, and spent a year in Europe with his parents. He prepped for a year at a boarding school at Lake Placid before entering Harvard in 1928, where he studied mathematical physics. After receiving a B.A. in 1932, he was awarded a Henry Fellowship to study at Cambridge University in England. Since by this time Garrett's interest had changed from mathematical physics to abstract algebra, he went to Munich to work on group theory.

Birkhoff returned to Harvard to become one of the first members of the Society of Fellows, an elite society of promising young scholars founded by President A. Lawrence Lowell in the early 30's as a kind of meta-PhD. Garrett taught at Harvard from 1933 until his retirement in 1981, and in 1969, he was appointed George Putnam Professor of Pure and Applied Mathematics. He reveled in the fact that he had no Ph.D., and if anyone forgot and addressed him as Dr. Birkhoff, he would sharply inform them that he was Meta. In the 1930s Birkhoff pioneered the study of lattice theory and established its importance in various mathematical disciplines. A lattice is a partially ordered set in which any two elements have a greatest lower bound and a least upper bound. In his 1940 book *Lattice Theory,* Birkhoff showed that various subjects, such as Boolean algebra, projective geometry, and affine geometry, could be treated as special types of lattices.

Garrett Birkhoff collaborated with Saunders MacLane in writing *Survey of Modern Algebra* (1941),

which became a standard college text. It assumed little more than an understanding of high school algebra and some geometry. The authors proceeded to develop the basic properties of central algebraic structures, including groups, rings and fields. The writing style was clear, concise and elegant. Each concept was fully defined and explained only when it was first needed. The proofs achieved an admirable balance of rigor and brevity. It was revised and reprinted in 1953 and 1965. The authors then developed a new text, *Algebra* (1967). In 1970, Birkhoff found a new collaborator, Thomas Bartee, which resulted in their *Modern Applied Algebra.* It was one of the earliest books to connect algebraic concepts to practical problems. It included finite groups, Boolean algebra, lattices, combinatorial analysis, and correlated algebraic coding theory.

Birkhoff wrote numerous books on applied mathematics, including *Hydrodynamics* (1950) and *Jets, Wakes and Cavities* (1957). In all, he published 229 papers, among them "On the Structure of Abstract Algebras" (1935), in which the definition of a Universal Algebra appeared for the first time. In the 1930s Garrett Birkhoff and John von Neumann pioneered quantum logic, which dealt with the foundations of quantum mechanics, and related to it, the behavior of finite, discrete deterministic systems. Throughout his career Birkhoff served as a consultant to military and industrial institutions, including the Los Alamos Scientific Laboratory and the Rand Corporation. During World War II he was involved in the development of the proximity fuse, which was meant to set off the explosion of anti-aircraft shells when they reached their targets.

While Garrett was never accused of being anti-Semitic, his extreme political conservatism stood out at Harvard, a bastion of "progressive" scholars. He was proudly politically incorrect in many of his social opinions. Despite what some viewed as grievous flaws and others as peccadilloes, for much of a century the Birkhoffs added great luster to Harvard's mathematics department, even though the son was occasionally confused with the father. Being the son of an internationally famous father is not an easy

role, but Garrett managed it rather well. He married Ruth Collins in 1938 and had three children. He died in his Water Mill, New York home.

Quotation of the day: “The primary service of modern mathematics is that it alone enables us to understand the vast abstract permanencies which underlie the flux of things.” – George David Birkhoff