Thomas Bayes

English theologian and mathematician **Thomas Bayes** (1702 – April 17, 1761) is believed to be the first to treat probability inductively. One of the founders of this discipline, Blaise Pascal, claimed that probability is only common sense reduced to calculation. Probability is the science of uncertainty. It is not known what is going to happen, only that in some situations it is believed that it is possible to assign some sort of measurement to various possible outcomes. Bayes established a mathematical basis for probability inference, that is, calculating the probability an event will occur in the future from the frequency that it has occurred in the past.

Born in London, Bayes was educated privately and like his father was ordained a nonconformist minister. At first he assisted his father, but in the late 1720's he took the position of minister at the Presbyterian Chapel in Tunbridge Wells, staying until he retired in 1752. Throughout his life he maintained a keen interest in mathematics, and most especially in the areas of probability and statistics. He was elected a member of the Royal Society after defending the views and philosophy of Sir Isaac Newton in a 1736 reply to the criticism of Bishop Berkeley.

In a now famous essay, "Towards Solving a Problem in the Doctrine of Chances," Bayes stated and solved the problem:

> "Given the number of times in which an unknown event has happened and failed:
> Required the chance that the probability of it happening in a single trial lies
> somewhere between any two degrees of probability that can be named."

Bayes' question is an example of an "inverse problem." If one knows that a certain event *E* occurs with probability p, one can calculate the probability that *E* occurs *m* times out of *n* trials. This is a "direct problem." Bayes turns it around and seeks the correct value of *p* (the probability of *E*) based on the results on *n* trials. The article was posthumously published in the *Philosophical Transactions of the Royal Society of London* by his friend Richard Price.

Bayes' work laid the foundation of modern Bayesian statistics, which attempts to calculate the probability of an event from a consideration of that event occurring together with a consideration of any relevant evidence. Pierre de Laplace accepted Bayes' conclusions in a memoir in 1781, but George Boole challenged them in his *The Laws of Thought* (1854). They have been controversial every since. Beginning in the 1950s, many statisticians advocated Bayesian methods as a solution for specific deficiencies found in standard statistical theory.

Bayes's definition of probability starts from a different basis than those who treat the word "probability" as a synonym for "frequency" or "fraction of events." The position of Bayesians is that probability measures "strength of belief." They argue that there is a subjective element in logical deduction and a probability assignment is a combination of what you believe and what the data tell you to believe. For a Bayesian, probability is defined as "our best possible approximation of what will actually occur." From this it is seen that Bayes regarded probability as a kind of betting ratio and tried to derive the laws of probability from this interpretation. He found a solution for his problem by means of an ingenious geometric representation. In his theory probabilities can be assigned only to events that either do or do not occur, otherwise a bet makes no sense.

If it is possible through beliefs about a hypothesis to assign probabilities to events and if an additional observation about the hypothesis whose degree of reliability is known or can be estimated is obtained, then it is possible to revise one's original probability distribution of events, thus reflecting the resulting beliefs given by the additional information. Bayes Theorem asserts that it is possible to use conditional probability to make predictions in reverse. That is, statistics collected on the occurrence of some event can be used to predict future similar occurrences. In sports, statistics of all kinds are kept, which help managers and coaches make decisions. In an article "Subconsciously, Athletes May Play Like Statisticians," which appeared in the *New York Times* on January 20, 2004, D. Leonhardt asserts that the Bayesian thought process is that which is used when "uncertainty becomes great enough to give past experience an edge over current observation."

The short form of Bayes theorem is used to find the conditional probability of event A given the probability of event B ($\neq 0$). A' is the complement of event A. The conditional probability of A given that event B occurred, symbolized by $P(A/B)$ is given by the formula:

$$P(A/B) = \frac{P(A) \cdot P(B/A)}{P(A) \cdot P(B/A) + P(A') \cdot P(B/A')}$$

The left-hand side of the equation is known as the "posterior probability." It is the adjusted probability of event A given that it is known that event B has occurred. The terms $P(A)$ and $P(A')$ are the probabilities of events A and A' respectively. P(A) is the probability assigned to event A prior to the introduction of the additional information B. It is called the "prior probability." $P(A')$ is the probability that A doesn't occur, that is $P(A') = 1 - P(A)$. The term $P(B/A)$ is the conditional probability of B given that A occurred, that is, the probability that the additional observation B would occur if the original event A happened. $P(B/A')$ is the probability of B given that A' occurred, that is, the probability that the additional observation occurred if it is know the event A did not occur.

As an illustration, suppose that there are two boxes, each containing computer printer cartridges. The first box has 30 black cartridges and 10 colored cartridges, while box two has 20 black and 20 colored.

If someone randomly chooses a box and then randomly chooses a cartridge from the box, what is the probability that the cartridge came from box one if it is black? Let A be the event that box one was chosen, then A' is the event that box one was not chosen, that is, box two was chosen. Let B be the event that the chosen cartridge was black. Since the box was chosen randomly, $P(A) = P(A') = ½$. From the contents of the boxes it is known that $P(B/A)$, the probability that a black cartridge was chosen given that box one was chosen is ¾ and $P(B/A')$, the probability that the cartridge was black given that box one was not chosen is ½. Substituting these values in the formula of Bayes theorem and we find that $P(A/B)$, the probability that the first box was chosen given that the cartridge is black, is 3/5.

Quotation of the Day: "I now send you an essay which I have found among the papers of our deceased friend Mr. Bayes, and which, in my opinion, has great merit…. In an introduction which he has writ to this Essay, he says, that his design at first in thinking of the subject of it was, to find out a method by which we might judge concerning the probability that an event has to happen, in given circumstances, upon supposition that we know nothing concerning it but that, under the same circumstances, it has happened a certain number of times, and failed a certain other number of times." – Richard Price, part of his letter to the Royal Society.