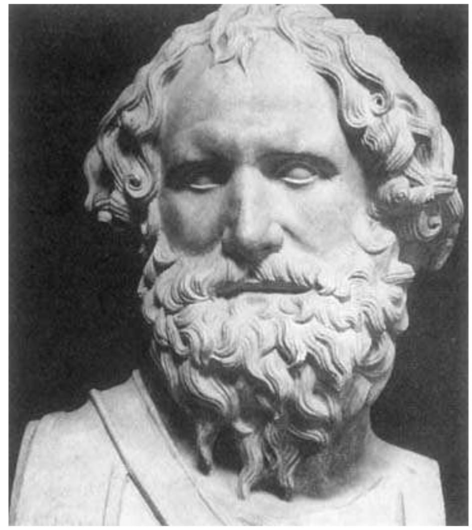
Archimedes



Archimedes (287 BCE – 212 BCE), the greatest mathematician and mathematical physicist of antiquity, was born in Syracuse in what is now Italy. His father Phidas was an astronomer. Archimedes probably studied in Alexandria with the followers of Euclid. Archimedes was the founder of the sciences of statics and hydrostatics, as well as being an ingenious engineer who used his talents to solve a wide range of practical problems. He also developed the principle of the lever and of multiple pulleys. He is credited with inventing a screw mechanism used to raise water from lower levels, but the Egyptians may have known of the Archimedean screw much earlier. His mathematical proofs were both boldly original and possessed rigor matching the highest standards of contemporary geometry. Much of what is known of his life is anecdotal, coming from the writings of historian, biographer and philosopher Plutarch, born 250 years after Archimedes' death.

During the second Punic War, Syracuse sided with Carthage against Rome and was besieged by Marcellus and the Roman troops for eight months during 212 BCE. Throughout the siege, both by land and by sea, Archimedes invented a range of ingenious ballistic machines that considerably delayed the capture of the city. According to Plutarch, he built huge catapults that hurled 500-pound boulders at the enemy and created large cranes with claw-like hooks that lifted ships out of the sea, shook out the sailors, and threw the vessels against large rocks, crushing them like eggs. It was asserted that he employed a series of mirrors that focused the sun on ships setting them afire, but this is now believed to be entirely fanciful. Reportedly, the Romans were so fearful of Archimedes' devices that once they were put to rout when he merely hung ropes over the wall. Many of the stories about Archimedes that have come down from secondary and tertiary sources may be myths as there is no firm historical evidence to support them.

After the city fell to Marcellus' troops, the general ordered that Archimedes was not to be harmed. Tragically, one of his soldiers came across an old man drawing figures in the sand. The soldier, who may not have known his captive's identity, ordered Archimedes to come with him. Archimedes responded, "Don't disturb my circles!" The enraged soldier slew the great geometer with his sword. Marcellus caused a splendid tomb to be constructed for Archimedes, and had engraved on it the figure of a sphere inscribed in a cylinder. This commemorated the proof Archimedes had given that the volume of a sphere is equal to two-thirds that of the circumscribing right cylinder and its surface to four times the area within a great circle.

Although Archimedes fame survived and spread through the world, his tomb and its monument were less fortunate. The people of Syracuse failed to honor their greatest citizen by keeping up his final resting place and before long it fell into neglect and its location forgotten. Some 137 years later, the great Roman orator Marcus Cicero was appointed an administrator in Sicily. He had learned of Archimedes while still a boy, and was fascinated by the man, whom he used as a model for "a good life." Cicero was successful in his mission of discovering Archimedes' tomb, which he described in his *Tusculanarum Disputationum*.

> "When I was quaestor in Sicily I managed to track down Archimedes' grave. The Syracusans knew nothing about it, and indeed denied that any such thing existed. But there it was, completely surrounded and hidden by bushes and brambles and thorns. I remembered having heard of some simple lines of verse which had been inscribed on his tomb, referring to a sphere and a cylinder modeled in stone on top of the grave. And so I took a good look round all the

numerous tombs that stand beside the Agrigentine Gate. Finally, I noticed a little column, just visible above the scrub: it was surmounted by a sphere and a cylinder. I immediately said to the Syracusans, some of whose leading citizens were with me at the time, that I believed this was the very object I had been looking for. Slaves were sent in with sickles to clear the site, and when a passage to the place was opened we approached the pedestal in front of us. The epigram was traceable with about half of the lines still legible, though approximately the second half of each line had been worn away."

Archimedes' achievements in the practical use of mathematics are legendary. School children are told the story of King Hieron's crown. The latter commissioned a goldsmith to make a crown of solid gold and provided the craftsman with the exact amount of the precious metal to make it. When the task was completed, the crown weighed the same as the gold allotted for its creation, but the king suspected the artisan substituted some silver in the piece and kept the extra gold for himself. Unable to prove it, he assigned the problem to Archimedes. While bathing, Archimedes realized that the weight of the water that overflowed his tub was equal to his body's weight when it was submerged. He was so excited; he jumped out of the tub, and ran naked to the king, shouting "Eureka!" ("I have found it.") The dishonest goldsmith was found out and punished.

For many the only thing learned from this account (if it actually happened) is that Archimedes was an early "streaker." They may be unaware of the importance and applications of the natural law of buoyancy, known as Archimedes' Principle, which states that a body that is completely or partially immersed in a fluid at rest is acted on by an upward, or buoyant, force, equal to the weight of the displaced fluid. The volume of fluid displaced is equal to the volume of the portion of the object submerged. The principle applies to floating and submerged bodies and to all fluids, including liquids and gases. It explains the buoyancy of ships and other vessels in water. For a ship to stay afloat, it does not require a great amount of water. It needs to be able to displace enough water to equal its weight. The principle also applies to the rise of a balloon or dirigible in the air. These objects float in the air due to the buoyant gas within the giant balloon-like bag.

Archimedes solved many problems put to him by his king and the king's son Gelon, but he dismissed these as trifles, being more interested in his purely theoretical work. In geometry, his extant works are *Measure of the Circle*, *Quadrature of the Parabola*, and *Spirals*. In *Measure* he concluded that the ratio of the circumference of a circle to its diameter is between 3 1/7 and 3 10/71. In the *Quadrature*, Archimedes used the method of exhaustion to find areas of regions bounded by geometric figures such as parabolas and ellipses. The method consists of finding sequences of regions whose areas can be calculated and are less than (or greater than) the area of a given region, and are increasing (or decreasing); then showing that the area approaches the area of the given region because the area between the boundary of the given region and the approximating region is "exhausted." His *Spirals* containing 28 propositions on the curve now known as the "spiral of Archimedes," including finding tangents to it. This spiral is traced by a point moving with uniform speed along a straight line that is revolving with uniform angular speed around a fixed point.

Archimedes' arithmetical works consisted of two papers. In the first, now lost, he devised a numeration system that would accommodate large numbers more easily than the unwieldy Greek number system, which used the letters of the alphabet. In the second paper, called *The Sand Reckoner*, he investigated very large numbers by determining the number of grains of sand required to fill the universe, constructing numbers up to 8×10^{17}. Later commentators reported that Archimedes made many astronomical observations. Unfortunately the book he wrote on the construction of a celestial sphere is lost, and so too is the sphere of the stars he constructed. Marcellus took this and Archimedes' orrery (a mechanical apparatus which illustrates with balls of various sizes the relative motions and positions of the bodies in the solar system), to Rome where they were preserved as curiosities for several hundred years and were seen by Cicero.

In a letter he sent to his friend Eratosthenes, Archimedes describes his *Method of Mechanical Theorems*. In this remarkable work, he reveals how he mechanically discovered his theorems before proving them rigorously and also explains his use of infinitesimals. What is particularly remarkable about the *Method* is that it contains the solution to some 20 problems using something akin to integration about 1900 years before Newton and Leibniz. It was believed that the text of the *Method* had been lost. But in 1907 Danish mathematical historian Johann Heiberg discovered a palimpsest at the Convent of the Holy Sepulcher in Istanbul. (A palimpsest is a book made on second-hand parchment, scrubbing out the original text and inking in the new.) This particular palimpsest originally was a scribe's copy in Greek of some of Archimedes' works, including the *Method*. Originally copied in the 10^{th} century, the document survived intact for 200 years with no one paying much attention to it and sometime between the 12^{th} and 14^{th} century, a monk looking to write a new prayer book, imperfectly washed out the Greek text on the pages, cut them in half, covered them with religious text and rebound them between boards. Heiberg discovered the faint Archimedean subtext, but the palimpsest disappeared or was stolen from the convent sometime during WWI.

Sometime in the 1930s, a French family acquired the book and in 1998 put it up for sale at Christie's in Manhattan. The Archimedes Palimpsest was sold for two million dollars to an anonymous buyer, who contacted The Walters Art Gallery in Baltimore for help in its restoration. The Walters asked the Rochester Institute of Technology and the Johns Hopkins University to each restore a few pages. Ultra-violet, infrared, and digital cameras were used to see through the later material to what remained of the original text and illustrations. Ultimately the entire text will be restored. Besides being the only copy of the *Method* in the original Greek, it also is the only Greek source of Archimedes' *On Floating Bodies*.

Reviel Netz, Professor of Ancient Science at Stanford University, summed up its value:

> "A manuscript is a window into the past. It allows us to get a view of a lost world. Some manuscripts provide us with an indirect view only, others with a better picture. What scholars do is put together all the evidence available, to form a single picture of the past. Our understanding of Archimedes is based on several manuscripts…. All of them, except the Archimedes Palimpsest, are very late copies which provide us only with a very indirect picture of Archimedes' mind. The best view we have, the closest we can get to Archimedes, is through the Palimpsest."

Quotation of the Day: "There are things which seem incredible to most men who have not studied mathematics." – Archimedes